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THE ITERATIVE RATIO METHOD APPLIED TO PARAMETER IDENTIFICATION

The task of finding the parameters of equations like $Nu = a\,Re^b Pr^c (Pr/Pr_w)^d$ by regression over an experimental set corresponding to a given heat exchanger, is difficult, without measuring the temperature on at least one side of the wall. There are two main problems: the implicit dependency on temperature of the bulk and near wall fluid and the impossibility of separating the cold and hot fluid partial heat transfer coefficients due to heat flow conservation. The classical way of zeroing the derivatives of a suitable objective function with respect to the parameters implies a multi–modal non–linear equation system, sensitive to the starting parameter vector. The new method starts with a given parameter vector and takes the ratio of hot over cold fluid partial heat transfer coefficient, separating them in this way. It then uses the total heat transfer equation to solve for each of them and the heat flow conservation equation to calculate the cold and hot side wall temperatures, returning thus to the initial equation. This one is used to re–compute the parameter vector. The whole process is iterated until satisfactory convergence occurs. The above mentioned method was compared with a parameter identification technique based on Genetic Algorithms.

The design, simulation or optimization of an apparatus, in which heat transfer between two fluids takes places with or without other processes, supposes good estimation of the partial heat transfer coefficients (PHTCs), for the specific hydrodynamic conditions.

One of the common equations used to describe the kinetics of the convective heat transfer from a fluid to a wall is (1), resulting from Buckingham analysis.

$$Nu = a\,Re^b\,Pr^c\left(\frac{Pr}{Pr_w}\right)^d \tag{1}$$

There are many situations when a substitute of equation (1) is used, namely (2), if one accepts the simplifying assumption of the independence of the physical properties with respect to temperature, except the viscosity.

$$Nu = a\,Re^b\,Pr^c\left(\frac{\mu}{\mu_w}\right)^d \tag{2}$$

The parameters a, b, c and d, characteristic for a given topology of the flow and heat transfer surface, should be computed by regression over an experimental set, sufficiently large to cover the domain of interest. Irrespective of the particular method used to determine these parameters, it is common practice to use two major types of experiments, in order to simplify the regression analysis. In the first one, either employ a uniform heat flow (measuring the temperature of the heat transfer surface in several points) or maintain the same surface temperature (steam in condensation). In the second one both fluids should have the same Reynolds numbers [1–6], in order to use, for both fluids, the same value for the partial heat transfer coefficient. Observing these experimental conditions it is easy to compute the PHTCs and then by linear regression, it is possible to determine the parameters of the aforementioned equation [7, 8]. If the experimental circumstances, such as a more complicated topology of the fluid paths, permit only the determination of the overall heat transfer coefficients (OHTCs), the computation of the parameters is possible only by non–linear regression, due to the lack of possibility to determine, directly from experiments, the PHTCs. The new proposed Iterative Ratio Method (IRM) allows the de–coupling of hot and cold fluids and partially eliminates the drawbacks of solving a non–linear system of equations.

Equations (1–2) are suitable to model heat transfer in different situations (distinct geometrical configurations of the apparatus, flow conditions etc.), thus their attractiveness. In these equations the physical properties are evaluated at the mean bulk fluid temperature, with the exception of viscosity, μ_w, or Prandtl number, Pr_w, which are evaluated at the wall temperature. The experimental data (flow rates and inlet/outlet temperatures of the fluids) permit only the calculation of the OHTCs, when the possibility of measuring the wall temperature is very difficult. The parameters of equations (1–2) are then determined by minimization of the objective function:

$$S = \sum_{i=1}^{N}\left[\frac{1}{\hat{k}_i} - \frac{1}{\alpha_{c,i}(a, b, c, d)} - \frac{1}{\alpha_{h,i}(a, b, c, d)} - \frac{\delta}{\lambda_w}\right]^2 \tag{3}$$

$$\alpha\,(a, b, c, d) = a\,Re^b\,Pr^c\left(\frac{Pr}{Pr_w}\right)^d\frac{\lambda}{d_{equ}}$$

In equation (3) $\hat{k}_i$ stands for the OHTC in experiment i. Minimizing equation (3) means solving a non–linear system of equations obtained by zeroing the derivatives of the aforementioned objective function with respect to parameters a, b, c and d:

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$$\begin{cases} \sum_{i=1}^{N} \left(\frac{1}{\alpha_{h,i}} + \frac{1}{\alpha_{c,i}}\right)\left(\frac{1}{\hat{k}_i} - \frac{1}{\alpha_{h,i}} - \frac{1}{\alpha_{c,i}} - \delta/\lambda_w\right) = 0 \\ \sum_{i=1}^{N} \left\{\left[\frac{\ln(Re_{h,i})}{\alpha_{h,i}} + \frac{\ln(Re_{c,i})}{\alpha_{c,i}}\right]\left(\frac{1}{\hat{k}_i} - \frac{1}{\alpha_{h,i}} - \frac{1}{\alpha_{c,i}} - \delta/\lambda_w\right)\right\} = 0 \\ \sum_{i=1}^{N} \left\{\left[\frac{\ln(Pr_{h,i})}{\alpha_{h,i}} + \frac{\ln(Pr_{c,i})}{\alpha_{c,i}}\right]\left(\frac{1}{\hat{k}_i} - \frac{1}{\alpha_{h,i}} - \frac{1}{\alpha_{c,i}} - \delta/\lambda_w\right)\right\} = 0 \\ \sum_{i=1}^{N} \left\{\left[\frac{\ln(Pr_{h,i}/Pr_{wh,i})}{\alpha_{h,i}} + \frac{\ln(Pr_{c,i}/Pr_{wc,i})}{\alpha_{c,i}}\right]\left(\frac{1}{\hat{k}_i} - \frac{1}{\alpha_{h,i}} - \frac{1}{\alpha_{c,i}} - \delta/\lambda_w\right)\right\} = 0 \end{cases} \quad (4)$$

Complementary, for each experimental set i, the equation system resulting from the heat flow conservation law must be solved, in order to obtain the wall temperature T_{wh} (for the hot fluid wall side) and T_{wc} (for the cold side):

$$\alpha_h (T_h - T_{wh}) = \frac{\lambda_w}{\delta}(T_{wh} - T_{wc}) = \alpha_c (T_{wc} - T_c) \quad (5)$$

The non–linear system (4–5) can only be solved by iterative methods, since it is impossible to explicit the unknowns. In the case of a great number of experimental data, N, the computational time is very long, because of the specific form of the system. Moreover, one can observe that, due to the non–linearity of the system, the final parameter values depend on the starting point, care should be taken to avoid local minima. Thus, many starting points should be used in order to attain the global minimum of the objective function.

THE ITERATIVE RATIO METHOD

Without the wall temperature, the experimental data obtained from a heat exchanger (the flow rates and inlet/outlet temperatures of the hot and cold fluids, assuming steady state) do not permit the de–coupling of hot and cold fluids and, thus, the computation of the PHTCs.

If the geometrical characteristics on both sides of the separation wall are the same (plate, spiral or twin helix heat exchangers, etc.) it may be assumed that the heat transfer equation applies equally for both fluids. If the flow regime is the same on both sides, so are the parameters of the model. When the flow regime is different, the parameters of the heat transfer model change accordingly.

So, if the flow regime for both fluids is the same, it is possible to write the ratio of the Nusselt numbers for both hot and cold fluids [7, 8]:

$$\frac{Nu_h}{Nu_c} = \left(\frac{Re_h}{Re}\right)^b \left(\frac{Pr_h}{Pr_c}\right)^c \left(\frac{Pr_h}{Pr_{wh}}\frac{Pr_{wc}}{Pr_c}\right)^d \quad (6)$$

which enables linking of the PHTCs of the fluids with a supplemental equation. Consequently, one may calculate the PHTC of the hot fluid (α_h), function of the cold one (α_cd) and the wall temperature on both sides, according to the following iterative procedure:

1. set a start vector for the parameters of the equation (1) $P^T = (a, b, c, d)$;

2. start the iteration counter, j = 1;

3. compute, for each experimental data set i, (steady–state measured values), the ratio:

$$r_i = \left(\frac{Re_{h,i}}{Re_{c,i}}\right)^b \left(\frac{Pr_{h,i}}{Pr_{c,i}}\right)^c \left[\left(\frac{Pr_{h,i}}{Pr_{wh,i}}\right) / \left(\frac{Pr_{c,i}}{Pr_{we,i}}\right)\right]^d \frac{\lambda_h}{\lambda_c} \quad (7)$$

where the index h is utilized for the hot fluid, and c for the cold one; at the first iteration, since the temperatures of the wall sides are unknown, we may assume that they are equal with the mean temperatures of the fluids (no resistance of the fluids to heat transfer);

4. compute, based on the heat flow conservation law, the PHTCs for both fluids and the wall temperatures on both sides (keep in mind the connection between the PHTCs made by the ratio r_i)

$$\alpha_{h,i} = \frac{1 + r_i}{\frac{1}{\hat{k}_i} - \frac{\delta}{\lambda_w}}$$

$$\alpha_{c,i} = \frac{\alpha_{h,i}}{r_i} \quad (8)$$

$$T_{wh,i} = \hat{T}_{h,i} - \frac{\hat{k}_i}{\alpha_{h,i}}(\hat{T}_{h,i} - \hat{T}_{c,i})$$

$$T_{we,i} = T_{wh,i} - \frac{\delta \cdot \hat{k}_i}{\lambda_w}(\hat{T}_{h,i} - \hat{T}_{c,i})$$

5. compute the new parameter vector, applying linear regression (take the natural logarithm of equation (1) and force all data to obey this heat transfer model):

$$P_c = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (X^T \cdot X)^{-1} \cdot (X^T \cdot B) \quad (9)$$

where the columns 2, 3 and 4 of the matrix of X are $\ln(Re_{h,i})/\ln(Re_{c,i})$, $\ln(Pr_{h,i})/\ln(Pr_{c,i})$, respectively, $\ln(Pr_{h,i}/Pr_{wh,i})/\ln(Pr_{c,i}/Pr_{wc,i})$, and the B vector is $\ln(Nu_{h,i})/\ln(Nu_{c,i})$;

6. compute the distance between P and P_c: $dist = \| P - P_c \|$;

7. if $dist < \varepsilon$ is suitably chosen, the iterative procedure stops. If not, the calculation is repeated from step 3, with new parameter values.

The application of the proposed method is quite simple, if the fluids are both in the same flow regime (so, for both of them, the parameters of equation (1) have the same values).

If the fluids flow in different regimes on both sides, the IRM is somewhat more difficult to apply, because the values of the parameters modify accordingly. This handicap may be eliminated if the experimental data are separated into regions, in accordance with the values of

the Reynolds number. The flow regime is verified first, at the beginning of a new iteration. When they are the same, equation (6) holds. If not, equation (6) becomes:

$$\frac{Nu_{h,L}}{Nu_{c,T}} = \frac{a_L}{a_T} \frac{Re_h^{b_L}}{Re_c^{b_T}} \frac{Pr_h^{b_L}}{Pr_c^{c_T}} \left(\frac{Pr_h}{Pr_{wh}}\right)^{d_L} \left(\frac{Pr_{wc}}{Pr_c}\right)^{d_T} \quad (10)$$

if the hot fluid is in a laminar regime and the cold one in a turbulent regime and conversely if it is the other way around.

RESULTS AND DISCUSSION

The IRM was applied to determine the PHTCs and the parameters of equation (1), which model the heat transfer in a twin helix heat exchanger [9]. The apparatus was operated with water as the cold fluid and water and diethyleneglycol as hot fluids. The flow rates were varied between 10 l/h and 200 l/h, and the inlet temperature of the hot fluid was kept constant at values between 22°C and 75°C. The experimental data (flow rates and inlet/outlet temperature measured at steady state) were used to compute the overall heat transfer coefficient and then the IRM was applied, as described above [9].

Figure 1 shows the dependency between the OHTC and the hot fluid Reynolds number, for the case under consideration.

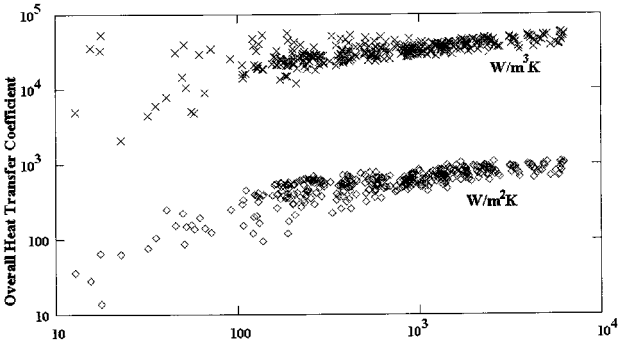

Figure 1 The dependency between the overall heat transfer coefficient and the hot fluid Reynolds number.

Implementation of the IRM, as detailed previously, was done in Mathcad, due to its versatility and possibility of writing re–usable code sequencing. Also, in Mathcad one can deal with simple formulas and equations, very few tokens being necessary.

Due to the very nature of the objective function (3), which is multi–modal by definition, several starting vectors were used, in order to achieve the global minimum. For the data set presented in Figure 1, the flow regime changes for the critical Reynolds number 2100, so a generalized relationship (10) was used in the calculations. Irrespective of the starting point, the following values were obtained for the parameters of the model, as described by equation (1):

- turbulent regime: a=0.0794, b=0.611, c=0.84 and d=0.156;
- laminar regime: a=0.759, b=0.447, c=0.45 and d=0.106

The departure model–experiment is presented in Figures 2 and 3 for the hot and cold fluid, respectively, the overall correlation coefficient being 0.968.

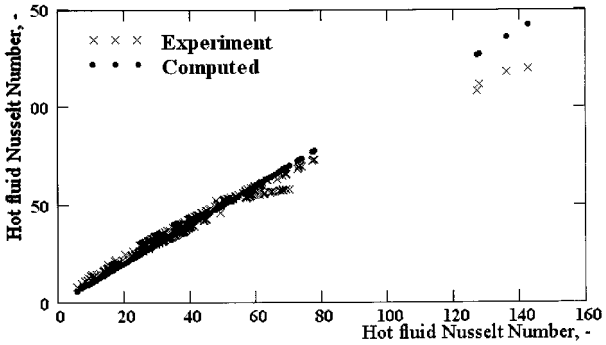

Figure 2 Parity plot of experimental and computed Nusselt numbers (equation 1) for the hot fluid.

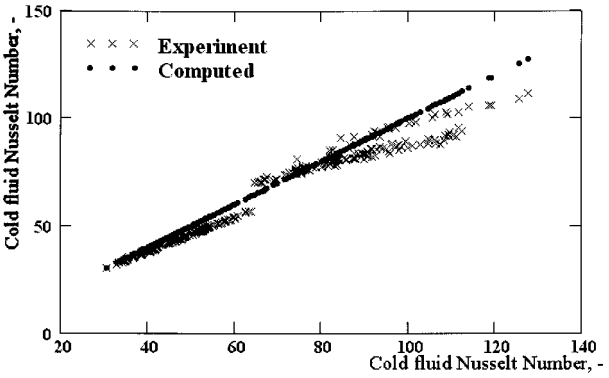

Figure 3 Parity plot of experimental and computed Nusselt numbers (equation 1) for the cold fluid.

To test whether or not the ratio Pr/Prw plays a significant role in modeling the heat transfer, as expressed by equation (1), we dropped this term from the model, simplifying it to:

$$Nu = aRe^b Pr^c \quad (11)$$

which means that we have only three parameters to determine by non–linear regression.

The IRM also worked well in this case, for the same data set the corresponding values for the parameters being:

- turbulent regime: a=0.08, b=0.615, c=0.94
- laminar regime: a=0.761, b=0.45, c=0.63

Remarkably, the whole temperature influence was taken over by the exponent of the Prandtl number. This is not surprising, since this term is the only one responsible for temperature in the model. The overall correlation coefficient was smaller, this time (only 0.923), due to the inherent restriction implied by removing one parameter. Also, in the new model, the wall temperature being left outside the computation, the model error increases for fluids having an important variation of viscosity with temperature.

In order to verify the accuracy of these solutions, obtained by the IRM, a check was made by solving the system given by equations (4–5) with an optimization technique based on Genetic Algorithms. The one point crossover probability was set to 0.6, while the mutation probability was 10^{-4}, values commonly accepted in the literature. The working populations had 1000 initial vectors and the allowed generation were up to 500. As expected, the result obtained for the parameters, whether or not the power of the group Pr/Prw was imposed, were the same as in the IRM case. The significant difference resides in the CPU time, which is much larger for Genetic Algorithms.

CONCLUSIONS

The paper presents an original algorithm to compute, by transforming non–linear regression into a linear one, the parameters of model (1), which describes a large spectrum of heat transfer equipment, that is the Iterative Ratio Method. The technique was applied to compute the parameters of the aforementioned model in the case of a twin helix heat exchanger, the hot fluid being water or ethyleneglycol and the cold fluid being water. The algorithm was implemented using the programming environment Mathcad, which is not a restriction. The obtained results are in good agreement with the literature. The major drawback of the IRM lies in the care that must be taken when each of the working fluids flows in a different regime. Nevertheless, the IRM proved to be reliable, since the same results were obtained with a program based on Genetic Algorithms.

REFERENCES

[1] S. Shin, Y.I. Cho, W.K. Gringrich, W Shyy, Int.J. Heat Mass Transfer **36**, (1993), p.4365–4373.

[2] C. J. Geancoplis, Transport Processes and Unit Operations, Allyn & Bacon, Boston, 1978.

[3] Gh. Jinescu, E. D. Lavric, in National Conference of Chemistry and Chemical Engineering, Ed. SIChR, (1993) vol. 2, October 29–30, Bucharest, p.473–477.

[4] Gh. Jinescu E. D., Lavric, S. Buligescu, Mathematical modeling of a highly compact heat exchanger, CHISA'96, (1996) 25–30 Aug., Prague, Czech Republic.

[5] W. T. Cross C., Ramshaw, Chem. Eng. Res. Des. **64**, (1986) p.293–301.

[6] K. Schubert, W. Bier, G. Linder, D. Seidel, Chem. Ing. Tech., **61** (1989), p.1–13.

[7] Gh. Jinescu, E. D. Lavric, National Conference of Chemistry and Chemical Engineering, Ed. SIChR, (1997) vol. I(1), October 16–18, Bucharest, p.109–114

[8] E. D. Lavric, V. Lavric, A new parameter identification technique: the iterative ratio method, CHISA'98, (1998) 23–28 Aug., Prague, Czech Republic.

[9] Gh. Jinescu, E. D. Lavric, V. Lavric, The new double helical heat exchanger. The mathematical modeling, CHISA'98, (1998) 23–28 Aug., Prague, Czech Republic.

REZIME

ODREĐIVANJE KONSTANTI U JEDNAČINI KRITERIJALNIH ZAVISNOSTI PRIMENOM ITERATIVNE METODE I ODNOSA KARAKTERISTIČNIH VELIČINA – PRORAČUN RAZMENJIVAČA TOPLOTE

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Često je veoma teško ispuniti zahtev koji se postavlja u pogledu određivanja parametara u nekoj jednačini, kao, na primer, u slučaju kriterijalne zavisnosti koja se koristi za definisanje prenosa toplote: $Nu = aRe^{b}Pr^{c}(Pr/Pr_w)^{d}$. Najčešće se u praksi primenjuje regresiona analiza i na osnovu eksperimentalnih podataka za odgovarajući razmenjivač toplote određuju vrednosti konstanti (parametara a–d) u kriterijalnoj zavisnosti. Međutim, da bi se odredile konstante podrazumeva se da su najmanje dostupni podaci o izmerenim temperaturama na jednoj strani zida koji deli topli od hladnog fluida. Dva glavna problema se pri tome pojavljuju: imlicitna zavisnost temperature fluida u glavnom toku (masi) i neposredno uz zid i nemogućnost da se razdvoje otpori (koeficijenti) za prelaz toplote sa strane hladnog i toplog fluida kao posledica zakona održanja prenosa toplote. Klasičan način koji se bazira na određivanju nula vrednosti prvog izvoda neke ciljne funkcije u odnosu na tražene parametre uključuje optimizaciju višeparametarske nelinearne regresije, koja zavisi od vektora početnih vrednosti parametara. Novi metod koji je razrađen u ovom radu zasniva se na nekoj početnoj vrednosti vektora parametra modela koji uzima u obzir odnos koeficijenata za prelaz toplote sa strane toplog, odnosno hladnog fluida. Poznavajući samo odnos koeficijenata, neophodno je iz ukupnog toplotnog bilansa da se odrede njihove pojedinačne vrednosti, a zatim na osnovu zakona o održanju prenosa toplote da se odrede temperature zida sa tople i hladne strane. Ove vrednosti se zatim koriste za ponovno izračunavanje vektora koji definiše parametre *a–d* u jednačini. $Nu = aRe^{b}Pr^{c}(Pr/Pr_w)^{d}$. Ceo postupak se ponavlja sve dok se na osnovu dva uzastopna određivanja parametara ne dobije zadovoljavajuća konvergencija. Predložen metod izračunavanja parametara je upoređen sa tehnikom određivanja koja se zasniva na primeni Genetskog Algoritma.

Key words: Iterative methods • Heat exchangers • Coiled tubes • Twin helix • Iterative ratio method •
Ključne reči: iterativna metoda • razmenjivači toplote • zmijasta cev • dvostruka zavojnica • iterativni metod odnosa •