CI&CEQ

Available on line at
Association of the Chemical Engineers of Serbia AChE
www.ache.org.rs/CICEQ

Chemical Industry & Chemical Engineering Quarterly 17 (4) 497–504 (2011) CI&CEQ

SRETEN B. STOJANOVIC[1]
DRAGUTIN LJ. DEBELJKOVIC[2]

[1]University of Niš, Faculty of Technology, Leskovac, Serbia
[2]University of Belgrade, Faculty of Mechanical Engineering, Department of Control Engineering, Belgrade, Serbia

SCIENTIFIC PAPER

UDC 621.3

DOI 10.2298/CICEQ110621035S

DELAY-DEPENDENT STABILITY ANALYSIS FOR DISCRETE-TIME SYSTEMS WITH TIME VARYING STATE DELAY

The stability of discrete systems with time-varying delay is considered. Some sufficient delay-dependent stability conditions are derived using an appropriate model transformation of the original system. The criteria are presented in the form of LMI, which are dependent on the minimum and maximum delay bounds. It is shown that the stability criteria are approximately the same conservative as the existing ones, but have much simpler mathematical form. The numerical example is presented to illustrate the applicability of the developed results.

Keywords: time-delay systems; interval time-varying delay; asymptotic stability; delay-dependent stability; Lyapunov method.

Time-delay frequently occurs in many practical systems, such as manufacturing systems, telecommunication and economic systems etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability.

A considerable attention has been paid to the problem of stability analysis and controller synthesis for continuous time-delay systems (see, *e.g.*, [1-15] and the reference therein). However, less attention has been drawn to the corresponding results for discrete-time delay systems (see, *e.g.*, [13,16-28]). This is mainly due to the fact that such systems can be transformed into augmented systems without delay. This augmentation of the system is, however, inappropriate for systems with unknown delays and for systems with time-varying delays, which are the subject analysis in this work. Furthermore, such an approach is not implementable because the dimension of the augmented system increases with the delay size; when the delay is large, the augmented system will become much complex and thus difficult to analyze and synthesize.

The existing stability conditions for time-delay systems can be classified into two types: delay-independent stability conditions and delay-dependent stability conditions. The former do not include any information about the magnitude of the delay, while the latter do employ such information. It is well known that delay-dependent stability conditions are generally less conservative than delay-independent ones, especially when the magnitude of the delay is small. Recently, increasing attention has been devoted to the problem of delay-dependent stability of linear systems with time-varying delay, including continuous-time [1--6,8-15] and discrete-time systems [14,17-28]. The key point for deriving the delay-dependent stability criterions is the choice of an appropriate Lyapunov-Krasovskii functional. It is known that the general form of this functional leads to a complicated system of partial differential equations, yielding infinite dimensional linear matrix inequalities (LMIs). That is why many authors have considered special forms of Lyapunov-Krasovskii functional and thus have derived simpler, but more conservative, sufficient conditions which can be represented by an appropriate set of linear matrix inequalities (LMIs).

In this paper, we propose new delay-dependent stability criteria, which dependent on the minimum and maximum delay bounds, for linear discrete systems with time-varying delays. By the Lyapunov function method, a delay-dependent criterion is derived in terms of linear matrix inequality (LMI), which can be solved efficiently by using standard optimisation tools. The special form of discrete-time counterpart Lyapunov-Krasovskii functional and novel techniques to achieve the delay dependence are used. Finally, a

Correspondening author: S.B. Stojanovic, University of Niš, Faculty of Technology, Leskovac, Serbia.
E-mail: ssreten@ptt.rs
Paper received: 21 June, 2011
Paper revised: 5 August, 2011
Paper accepted: 15 August, 2011

numerical example is included to show that our results are approximately the same conservative as the existing ones [23], but have much simpler mathematical presentation. Our stability condition is expressed by means of one LMI, while the stability condition in [23] is presented by two LMI.

Notation. $\Re^n$ and Z^+ denote the *n*-dimensional Euclidean space and positive integers. Notation $\mathbf{P} > 0$ ($\mathbf{P} \geq 0$) means that matrix $\mathbf{P}$ is real, symmetric and positive definite (semi-definite). For real symmetric matrices $\mathbf{P}$ and $\mathbf{Q}$, the notation $\mathbf{P} > \mathbf{Q}$ ($\mathbf{P} \geq \mathbf{Q}$) means that matrix $\mathbf{P} - \mathbf{Q}$ is positive definite (positive semi-definite). *I* is an identity matrix with an appropriate dimension. Superscript "*T*" represents the transpose. In symmetric block matrices or complex matrix expressions, we use an asterisk (*) to represent a term which is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

PROBLEM FORMULATION AND SOME PRELIMINARIES

Consider a linear discrete-time varying delay in the state:

$$x(k+1) = Ax(k) + Bx(k - h(k)) \tag{1}$$

with an associated function of initial state:

$$x(\theta) = \psi(\theta), \theta \in \{-h_M, -h_M + 1, \ldots, 0\} \triangleq \Delta \tag{2}$$

where $x(k) \in \Re^n$ is the state at instant *k*, matrices $A \in \Re^{n \times n}$ and $B \in \Re^{n \times n}$ are constant matrices. *h*(*k*) is the positive integer representing the time delay of the system that we assume to be time dependent and satisfies the following:

$$h_m \leq h(k) \leq h_M \tag{3}$$

where h_m and h_M and are constant positive integer representing the minimum and maximum delays, respectively.

The assumption on the time delay *h*(*k*) in Inequality (3) characterises the real situation in many practical applications. A typical example containing time delays that can be characterised by Inequality (3) can be found in networked control system, where the delays induced by the network transmission (either from sensor to controller or from controller to actuator) are actually time-varying, and can be assumed to have minimum and maximum delay bounds without loss of generality.

Let $D(\Delta, \Re^n)$ is space of functions mapping the discrete interval Δ into $\Re^n$. Then, for $\phi(\theta) \in D$:

$$\|\phi\|_D \triangleq \sup_{\theta \in \Delta} \|\phi(\theta)\| \tag{4}$$

is the norm of an element $\phi \in D$ in space *D*.

Let $\Re^\gamma = \{\phi \in D : \|\phi\|_D < \gamma, \gamma \in \Re\} \subset D$.

Definition 1 [16]. The equilibrium state $x = 0$ of system given by Eq. (1) is asymptotically stable if any initial $\psi(\theta)$ which satisfies:

$$\psi(\theta) \in D^\infty \tag{5}$$

holds:

$$\lim_{k \to \infty} \mathbf{x}(k, \psi) \to 0 \tag{6}$$

Lemma 1 [16]. If there exist positive numbers α and β and continuous functional $V : D \to \Re$ such that

$$0 < V(x_k) \leq \alpha \|x_k\|_D^2, \quad \forall x_k \neq 0, \quad V(0) = 0 \tag{7}$$

$$\Delta V(x_k) \triangleq V(x_{k+1}) - V(x_k) \leq -\beta \|x(k)\|^2 \tag{8}$$

$\forall x_k \in D$ satisfying Eq. (1), then the solution $x = 0$ of Eqs. (1) and (2) is asymptotically stable.

Definition 2 [16]. Discrete system with time delay given by Eq. (1) is asymptotically stable if and only if its solution $x = 0$ is asymptotically stable.

MAIN RESULTS

In this section, we aim to establish an asymptotic stability criterion for the system given by Eq. (1) using the Lyapunov method combined with the linear matrix inequality (LMI) technique. Unlike [23], in this paper the transformation model is only partly used for the getting criteria. Our results are approximately the same conservative as the existing ones in literature [23], but have much simpler mathematical presentation. Namely, our stability condition is expressed by means of one LMI, while the stability condition in [23] is presented by two LMI.

Theorem 1. System given by Eq. (1) is asymptotically stable for all *h* satisfying Inequality (3) if there exist real symmetric matrices $P > 0$, $Q > 0$ and $Z > 0$ satisfying the following LMI:

$$\Omega = \begin{bmatrix} \Gamma & (A+B)^T PB & -A^T PB & h_M (A-I)^T Z \\ * & -Q & -B^T PB & h_M B^T Z \\ * & * & -Z & 0 \\ * & * & * & -Z \end{bmatrix} < 0 \tag{9}$$

$$\Gamma = -P - P^T + (h_M - h_m + 1)Q + A^T P(A+B) + \\ + (A+B)^T PA$$

Proof. Let

$$\begin{aligned}\eta(m) &\triangleq x(m+1)-x(m)\\ &= Ax(m)+Bx\left(m-h(m)\right)-x(m)\\ &= (A-I)x(m)+Bx\left(m-h(m)\right)\end{aligned} \tag{10}$$

and

$$\begin{aligned}x\left(k-h(k)\right) &= x(k)-\left(x(k)-x\left(k-h(k)\right)\right)\\ &= x(k)-\sum_{m=k-h(k)}^{k-1}\left(x(m+1)-x(m)\right)\\ &= x(k)-\sum_{m=k-h(k)}^{k-1}\eta(m)\end{aligned} \tag{11}$$

Then, the system given by Eq. (1) can be transformed into:

$$\begin{aligned}x(k+1) &= Ax(k)+Bx(k-h(k)\\ &= Ax(k)+B\left(x(k)-\sum_{m=k-h(k)}^{k-1}\eta(m)\right)\end{aligned}$$

i.e.

$$x(k+1)=(A+B)x(k)-B\sum_{m=k-h(k)}^{k-1}\eta(m) \tag{12}$$

Choose a discrete-time counterpart Lyapunov--Krasovskii functional candidate as:

$$\begin{aligned}V(k) &= V_1(k)+V_2(k)+V_3(k)+V_4(k)\\ V_1(k) &= x^T(k)Px(k)\\ V_2(k) &= \sum_{i=k-h(k)}^{k-1}x^T(i)Qx(i)\\ V_3(k) &= \sum_{j=-h_M+2}^{-h_m+1}\sum_{i=k+j-1}^{k-1}x^T(i)Qx(i)\\ V_4(k) &= \sum_{i=-h_M}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j)Z\eta(j)\end{aligned} \tag{13}$$

where P, Q and Z are positive definite matrices to be determined.

Taking the forward difference:
$\Delta V = V(k+1)-V(k)$,
along the solutions of system given by Eq. (1) and transformed system given by Eq. (12), we can obtain:

$$\begin{aligned}\Delta V_2(k) &= \sum_{i=k+1-h(k+1)}^{k}x^T(i)Qx(i)-\sum_{i=k-h(k)}^{k-1}x^T(i)Qx(i)\\ &= \sum_{i=k+1-h(k+1)}^{k-1}x^T(i)Qx(i)+x^T(k)Qx(k)-\\ &-\sum_{i=k-h(k)}^{k-1}x^T(i)Qx(i)+\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)-\\ &-\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)=x^T(k)Qx(k)-\\ &-x^T\left(k-h(k)\right)Qx\left(k-h(k)\right)\\ &+\sum_{i=k+1-h(k+1)}^{k-1}x^T(i)Qx(i)-\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)\end{aligned} \tag{15}$$

Whereas:

$$h(k+1)\le h_M,\quad h(k)\ge h_m \tag{16}$$

then:

$$\begin{aligned}&\sum_{i=k+1-h(k+1)}^{k-1}x^T(i)Qx(i)=\sum_{i=k+1-h(k+1)}^{k-h_m}x^T(i)Qx(i)+\\ &+\sum_{i=k-h_m+1}^{k-1}x^T(i)Qx(i)\le\sum_{i=k+1-h_M}^{k-h_m}x^T(i)Qx(i)+\\ &+\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)\end{aligned} \tag{17}$$

Therefore:

$$\begin{aligned}\Delta V_2(k) &= x^T(k)Qx(k)-x^T\left(k-h(k)\right)Qx\left(k-h(k)\right)\\ &+\sum_{i=k+1-h(k+1)}^{k-1}x^T(i)Qx(i)-\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)\\ &\le x^T(k)Qx(k)-x^T\left(k-h(k)\right)Qx\left(k-h(k)\right)\\ &+\sum_{i=k+1-h_M}^{k-h_m}x^T(i)Qx(i)+\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)-\\ &-\sum_{i=k+1-h(k)}^{k-1}x^T(i)Qx(i)=x^T(k)Qx(k)-\\ &x^T\left(k-h(k)\right)Qx\left(k-h(k)\right)+\sum_{i=k+1-h_M}^{k-h_m}x^T(i)Qx(i)\end{aligned} \tag{18}$$

$$\begin{aligned}&\Delta V_1(k)=x^T(k+1)Px(k+1)-x^T(k)Px(k)=\left[Ax(k)+Bx(k-h(k))\right]^T\left[P(A+B)x(k)-B\sum_{m=k-h(k)}^{k-1}\eta(m)\right]-\\ &-x^T(k)Px(k)=x^T(k)\left[A^TP(A+B)-P\right]x(k)+x^T(k)\left[-A^TPB\right]\sum_{m=k-h(k)}^{k-1}\eta(m)+x^T(k-h(k))B^TP(A+B)x(k)+\\ &+x^T(k-h(k))\left[-B^TPB\right]\sum_{m=k-h(k)}^{k-1}\eta(m)\end{aligned} \tag{14}$$

In addition:

$$\begin{aligned}
\Delta V_3(k) &= \sum_{j=-h_M+2}^{-h_m+1} \sum_{i=k+j}^{k} x^T(i)Qx(i) - \\
&- \sum_{j=-h_M+2}^{-h_m+1} \sum_{i=k+j-1}^{k-1} x^T(i)Qx(i) = \\
&= \sum_{j=-h_M+2}^{-h_m+1} \begin{pmatrix} x^T(k)Qx(k) - \\ -x^T(k-1+j)Qx(k-1+j) \end{pmatrix} \\
&= \left(-h_m+1-(-h_M+2)+1\right) x^T(k)Qx(k) - \\
&- \sum_{j=-h_M+2}^{-h_m+1} x^T(k-1+j)Qx(k-1+j) = \\
&= (h_M-h_m)x^T(k)Qx(k) - \sum_{i=k-h_M+1}^{k-h_m} x^T(i)Qx(i)
\end{aligned} \tag{19}$$

Finally:

$$\begin{aligned}
\Delta V_4(k) &= \sum_{i=-h_M}^{-1} \sum_{j=k+1+i}^{k} \eta^T(j)Z\eta(j) - \\
&- \sum_{i=-h_M}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)Z\eta(j) = \\
&= \sum_{i=-h_M}^{-1} \left(\eta^T(k)Z\eta(k) - \eta^T(k+i)Z\eta(k+i)\right) \\
&= h_M\eta^T(k)Z\eta(k) - \sum_{i=-h_M}^{-1} \eta^T(k+i)Z\eta(k+i) \\
&= h_M\eta^T(k)Z\eta(k) - \sum_{m=k-h_M}^{k-1} \eta^T(m)Z\eta(m) \\
&\le h_M\eta^T(k)Z\eta(k) - \sum_{m=k-h(k)}^{k-1} \eta^T(m)Z\eta(m)
\end{aligned} \tag{20}$$

From Eqs. (14) and (18)-(20), we have:

$$\begin{aligned}
\Delta V(k) &= \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) \\
&\le x^T(k)\left[A^TP(A+B)-P\right]x(k) + \\
&+ x^T(k)\left[-A^TPB\right] \sum_{m=k-h(k)}^{k-1} \eta(m) + \\
&+ x^T(k-h(k))B^TP(A+B)x(k) + \\
&+ x^T(k-h(k))\left[-B^TPB\right] \sum_{m=k-h(k)}^{k-1} \eta(m) + x^T(k)Qx(k) - \\
&- x^T(k-h(k))Qx(k-h(k)) + \sum_{i=k+1-h_M}^{k-h_m} x^T(i)Qx(i) + \\
&+ (h_M-h_m)x^T(k)Qx(k) - \sum_{i=k-h_M+1}^{k-h_m} x^T(i)Qx(i) + \\
&+ h_M\eta^T(k)Z\eta(k) - \sum_{m=k-h(k)}^{k-1} \eta^T(m)Z\eta(m)
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Delta V(k) &\le x^T(k)[A^TP(A+B)-P + \\
&+ (d_M-d_m+1)Q]x(k) + x^T(k)\left[-A^TPB\right] \\
&\times \sum_{m=k-h(k)}^{k-1} \eta(m) + x^T(k-h(k))B^TP(A+B)x(k) + \\
&+ x^T(k-h(k))\left[-B^TPB\right] \sum_{m=k-h(k)}^{k-1} \eta(m) + \\
&+ x^T(k-h(k))\times(-Q)x(k-h(k)) + \\
&+ h_M\left[(A-I)x(k) + Bx(k-h(k))\right]^T Z[(A-I)x(k) + \\
&+ Bx(k-h(k))] - \sum_{m=k-h(k)}^{k-1} \eta^T(m)Z\eta(m)
\end{aligned} \tag{22}$$

Note that:

$$\begin{aligned}
&\sum_{m=k-h(k)}^{k-1} \eta^T(m)Z\eta(m) \ge \frac{1}{h(k)} \left(\sum_{m=k-h(k)}^{k-1} \eta(m) \right)^T \times \\
&\times Z \sum_{m=k-h(k)}^{k-1} \eta(m) \ge \frac{1}{h_M} \left(\sum_{m=k-h(k)}^{k-1} \eta(m) \right)^T \times \\
&\times Z \sum_{m=k-h(k)}^{k-1} \eta(m)
\end{aligned} \tag{23}$$

Therefore, we have:

$$\begin{aligned}
\Delta V(k) &\le x^T(k)[A^TP(A+B)-P + \\
&+ (h_M-h_m+1)Q + h_M(A-I)^TZ(A-I)x(k) + \\
&+ 2x^T(k)\left[\frac{1}{2}(A+B)^TPB + h_M(A-I)^TZB\right] \times \\
&\times x(k-h(k)) + 2x^T(k)\left[-\frac{1}{2}A^TPB\right] \sum_{m=k-h(k)}^{k-1} \eta(m) + \\
&+ x^T(k-h(k))(-Q + h_MB^TZB)x(k-h(k)) + \\
&+ 2x^T(k-h(k))\left[-\frac{1}{2}B^TPB\right] \sum_{m=k-h(k)}^{k-1} \eta(m) + \\
&+ \left(\sum_{m=k-h(k)}^{k-1} \eta(m) \right)^T \frac{-1}{h_M} Z \sum_{m=k-h(k)}^{k-1} \eta(m)
\end{aligned} \tag{24}$$

i.e.,

$$\Delta V(k) \le \xi^T(k)\Omega\xi(k) \tag{25}$$

where:

$$\xi(k) = \left[x^T(k) \;\; x^T(k-h(k)) \;\; \sum_{m=k-h(k)}^{k-1} \eta^T(m) \right]^T$$

$$\hat{\Omega} = \left[\begin{array}{c|c|c} \hat{\Gamma} & \frac{1}{2}(A+B)^TPB + h_M(A-I)^TZB & -\frac{1}{2}A^TPB \\ \hline * & -Q + h_MB^TZB & -\frac{1}{2}B^TPB \\ \hline * & * & -\frac{1}{h_M}Z \end{array} \right] \tag{26}$$

$$\begin{aligned}
\hat{\Gamma} &= A^TP(A+B) - P + (h_M-h_m+1)Q + \\
&+ h_M(A-I)^TZ(A-I)
\end{aligned}$$

Obviously, $\Delta V(k)<0$ if $\Omega<0$. Introducing a substitution $P \leftrightarrow 2P$, we have:

$$\tilde{\Omega}=\left[\begin{array}{c|c|c} \tilde{\Gamma} & (A+B)^T PB+h_M(A-I)^T ZB & -A^T PB \\ \hline * & -Q+h_M B^T ZB & -B^T PB \\ \hline * & * & -\frac{1}{h_M}Z \end{array}\right]<0 \tag{27}$$

$$\tilde{\Gamma}=-P-P^T+A^T P(A+B)+(A+B)^T PA+$$
$$+(h_M-h_m+1)Q+h_M(A-I)^T Z(A-I)$$

Further:

$$\left[\begin{array}{c|c|c} \tilde{\Gamma} & (A+B)^T PB+h_M(A-I)^T ZB & -A^T PB \\ \hline * & -Q+h_M B^T ZB & -B^T PB \\ \hline * & * & -\frac{1}{h_M}Z \end{array}\right]=$$

$$=\left[\begin{array}{c|c|c} \Gamma & (A+B)^T PB & -A^T PB \\ \hline * & -Q & -B^T PB \\ \hline * & * & -\frac{1}{h_M}Z \end{array}\right]+$$

$$+\left[\begin{array}{c|c|c} h_M(A-I)^T Z(A-I) & h_M(A-I)^T ZB & 0 \\ \hline * & h_M B^T ZB & 0 \\ \hline * & * & 0 \end{array}\right]= \tag{28}$$

$$=\left[\begin{array}{c|c|c} \Gamma & (A+B)^T PB & -A^T PB \\ \hline * & -Q & -B^T PB \\ \hline * & * & -\frac{1}{h_M}Z \end{array}\right]+$$

$$+\begin{bmatrix} (A-I)^T Z \\ B^T Z \\ 0 \end{bmatrix}\left(-\frac{1}{h_M}Z\right)^{-1}\begin{bmatrix} Z(A-I) & ZB & 0 \end{bmatrix}<0$$

where:

$$\Gamma=\tilde{\Gamma}-h_M(A-I)^T Z(A-I)=-P-P^T+$$
$$+A^T P(A+B)+(A+B)^T PA+(h_M-h_m+1)Q \tag{29}$$

Using Schur complements, it is easy to see that the condition given by Inequality (28) is equivalent to:

$$\left[\begin{array}{c|c|c|c} \Gamma & (A+B)^T PB & -A^T PB & (A-I)^T Z \\ \hline * & -Q & -B^T PB & B^T Z \\ \hline * & * & -\frac{1}{h_M}Z & 0 \\ \hline * & * & * & -\frac{1}{h_M}Z \end{array}\right]<0 \tag{30}$$

Introducing a substitution $h_M Z \leftrightarrow Z$, we obtain the LMI given by Inequality (9). From this, it follows that the Inequality (9) guarantees that $\Delta V(k)<0$ for all non-zero $\xi(k)$. Hence, the system given by Eq. (1) is asymptotically stable for all time-varying delay $h(k)$ satisfying Inequality (3).

Remark 1. The condition given by Inequality (9) in Theorem 1 is LMI condition, therefore can be easily checked by using standard numerical software. This condition depends on both the maximum and minimum delay bounds. In [23] the stability condition is defined by two LMI and numerically more complex.

Remark 2. The Lyapunov-Krasovskii functionals defined in this work (Theorem 1) and [23] (Theorem 1) are mutually identical. However, they differ with respect to the forward difference $\Delta V=V(k+1)-V(k)$. For performing the forward difference [23] in a model transformation given in Eq. (12) is used twice, whereas in our paper used only once (see Eq. (14)).

Remark 3. From Theorem 1, for the constant delay case ($h_M=h_m=h$), we have the next result.

Corollary 1. System given by Eq. (1) is asymptotically stable with $h(k) = h$ if there exist real symmetric matrices $P > 0$, $Q > 0$ and $Z > 0$ satisfying the following LMI:

$$\Phi=\begin{bmatrix} \mathrm{T} & (A+B)^T PB & -A^T PB & h(A-I)^T Z \\ * & -Q & -B^T PB & hB^T Z \\ * & * & -Z & 0 \\ * & * & * & -Z \end{bmatrix}<0 \tag{31}$$

$$\mathrm{T}=-P-P^T+Q+A^T P(A+B)+(A+B)^T PA$$

Proof. From the condition given by Inequality (9), for $h_M=h_m=h$, follows the condition given by Inequality (31).

RESULTS AND DISCUSSION

In this section, one example is used to demonstrate that the method presented in this paper is effective in implementation.

Example 1. Consider the following two discrete-time systems with time-varying delay in the state:

S1:

$$x(k+1)=\begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}x(k)+\begin{bmatrix} -0.2 & 0 \\ -0.4 & -0.1 \end{bmatrix}x(k-h(k)),$$

$$x(k)=\begin{bmatrix} x_1(k) & x_2(k) \end{bmatrix}^T$$

S2:

$$x(k+1)=\begin{bmatrix} 0.6 & 0 \\ 0.35 & 0.7 \end{bmatrix}x(k)+\begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}x(k-h(k)),$$

$$x(k)=\begin{bmatrix} x_1(k) & x_2(k) \end{bmatrix}^T$$

Let first determine the upper limit of delay h_M for $h_m \in \{0,1,2,3\}$, so that the systems S1 and S2 are asymptotically stable. Using Theorem 1 and Theorem 1 from [23], the upper limit of delay h_M for given h_m are calculated and shown in Table 1. It is clear that the obtained results are almost identical. However, the advantage of our results compared to the results presented in [23] is the simpler mathematical and numerical form. Namely, in Theorem 1 the stability condition is expressed by means of one LMI, while the stability condition (Theorem 1 in [23]) is presented by two LMI.

Also, using Corollary 1, the upper limit of constant delay h is calculated and shown in Table 1, so that the systems S1 and S2 are yet stable. It is obvious that Corollary 1 gives wider range of delay. This is because the Corollary 1 assumes that the delay is constant, as opposed to Theorem 1 which takes into account the time-varying delay.

In order to verify previous results, the system operation is simulated under the following conditions:

S1:

time-delay: a) $h(k) = h = 5$ or b) $2 \le h(k) \le 5$ is random integer variable.

Initial state:

$$x(\theta) = \psi(\theta) = \begin{bmatrix} 3 & 4 & 5 & 5 & 4 & 3 \\ -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix}^T, \quad \theta \in \{-5, -4, \dots, 0\}$$

S2:

time-delay: a) $h(k) = h = 10$ or b) $2 \le h(k) \le 10$ is random integer variable.

Initial state:

$$x(\theta) = \psi(\theta) = \begin{bmatrix} 5 & 4 & 3 & 2 & 2 & 3 & 4 & 5 & 5 & 4 & 3 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix}^T, \quad \theta \in \{-10, -9, \dots, 0\}$$

Figures 1 and 2 show the initial condition response of the systems S1 with $h(k) = h = 5$ and S2 with $h(k) = h = 10$. It is observed that the values of

Table 1. Upper bounds of time delay

System	Theorem 1 [23]	Theorem 1, this work	Corollary 1
S1	$0 \le h(k) \ge 4$	$0 \le h(k) \ge 4$	$h \le 11$
	$1 \le h(k) \ge 4$	$1 \le h(k) \ge 4$	
	$2 \le h(k) \ge 3$	$2 \le h(k) \ge 5$	
	$3 \le h(k) \ge 5$	$3 \le h(k) \ge 5$	
S2	$0 \le h(k) \ge 10$	$0 \le h(k) \ge 10$	$h \le 19$
	$1 \le h(k) \ge 11$	$1 \le h(k) \ge 11$	
	$2 \le h(k) \ge 12$	$2 \le h(k) \ge 11$	
	$3 \le h(k) \ge 13$	$3 \le h(k) \ge 11$	

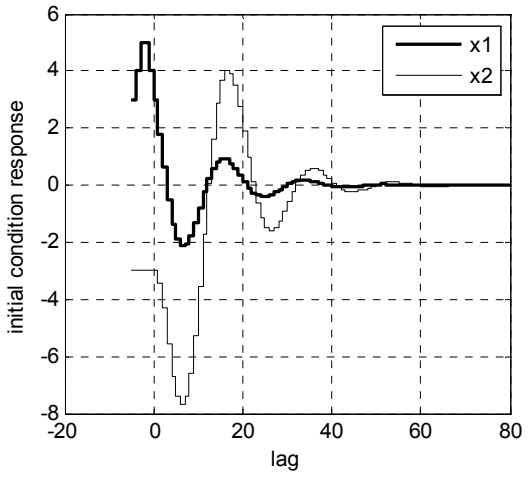

Figure 1. Initial condition response of the system S1 with constant time-delay $h(k) = h = 5$.

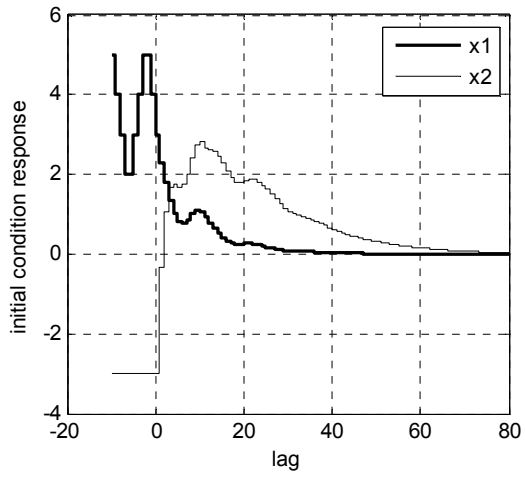

Figure 2. Initial condition response of the system S2 with constant time-delay $h(k) = h = 10$.

state variables $x \to 0$ when $t \to \infty$, which proves that the systems S1 and S2 are asymptotically stable.

The initial condition response of the systems S1 with time-varying delay and sequence time-varying delay $2 \le h(k) \le 5$ as random integer variable are shown in Figures 3 and 4. From Figure 3, it is also observed that when $t \to \infty$, the values of state variables $x \to 0$, which proves that the system S1 is asymptotically stable.

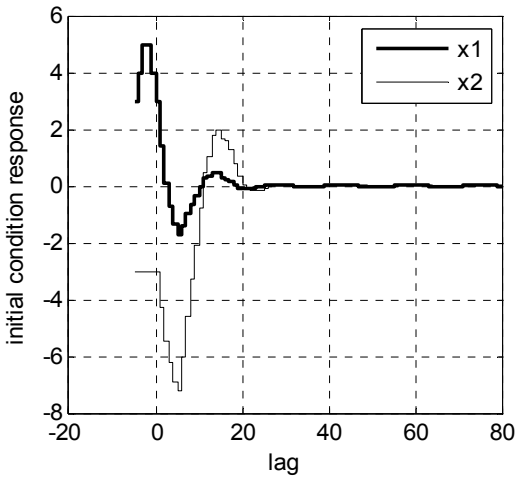

Figure 3. Initial condition response of the system S1 with time-varying delay 2≤h(k)≤5.

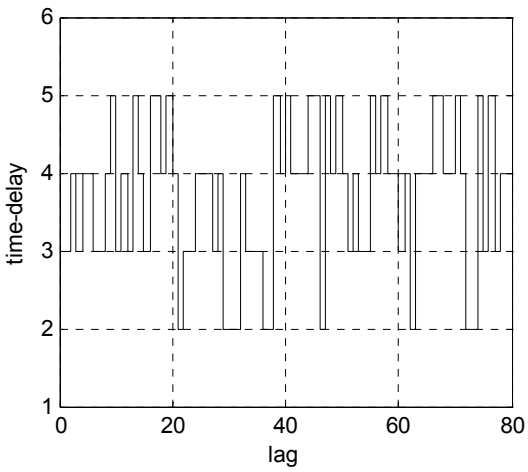

Figure 4. Time-varying delay is random integer variable 2≤h(k)≤5..

Figures 5 and 6 show the initial condition response of the systems S2 with time-varying delay and sequence time-varying delay $2 \le h(k) \le 10$. as random integer variable. From Figure 5, it can be also concluded that the system S2 is asymptotically stable.

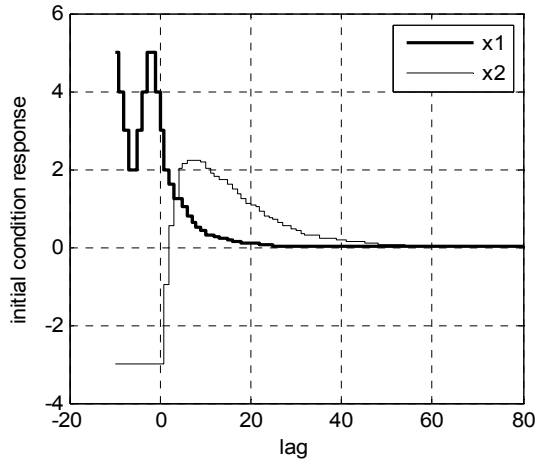

Figure 5. Initial condition response of the system S2 with time-varying delay 2≤h(k)≤10..

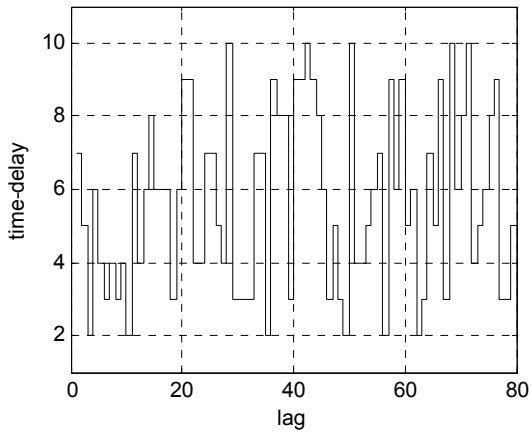

Figure 6. Time-varying delay is random integer variable $h(k) = h = 10$.

CONCLUSION

In this paper, the problem of delay-dependent stability for discrete time systems with time-varying state delay is investigated. Some new LMI sufficient conditions are proposed, which is dependent of the minimum and maximum delay bounds. This stability condition can be easily verified *via* standard numerical software. It was shown that the results coincide with the existing results from the literature. The advantages of our results, compared to the previously published results, are the simpler mathematical form and the numerical efficiency. Namely, in Theorem 1, we express the stability condition by means of one LMI, while previously two LMIs have been employed. Finally, a numerical example is included to show that our results are approximately the same conservative as the existing ones in the literature.

Acknowledgment

Parts of this research were supported by the Ministry of Education and Sciences of Republic of Serbia through Mathematical Institute SANU Belgrade, Grant ON174001, Dynamics of hybrid systems with complex structures, Mechanics of materials, and Faculty of Technology, University of Niš.

REFERENCES

[1] O.M. Kwon, J.H. Park, S.M. Lee, J. Optimiz. Theory App. **145** (2010) 343-353

[2] O.M. Kwon, J.H. Park, S.M. Lee, Appl. Math. Comput. **203** (2008) 937-942

[3] E. Fridman, U. Shaked, IEEE Trans. Autom. Control **47** (2002) 253-270

[4] E. Fridman, Syst. Control Lett. **43** (2001) 309-319

[5] Q.L. Han, D. Yue, IET Control Theory Appl. **1** (2007) 854--859

[6] Y. He, M. Wu, J.H. She, G.P. Liu, IEEE Trans. Autom. Control **49** (2004) 828-832

[7] Y.S. Moon, P. Park, W.H. Kwon, Automatica **37** (2001) 307-312

[8] P. Park, J.W. Ko, Automatica **43** (2007) 1855-1858

[9] E. Shustin, E. Fridman, Automatica **43** (2007) 1649-1655

[10] M. Wu, Y. He, J.H. She, G.P. Liu, Automatica **40** (2004) 1435-1439

[11] S. Xu, J. Lam, IEEE Trans. Autom. Control **50** (2005) 384-387

[12] D. Yue, Q.L. Han, IEEE Trans. Circuits Syst. II **51** (2004) 685-689

[13] D. Yue, E. Tian, Y. Zhang, Int. J. Robust Nonlinear Control **19** (2009) 1493-1518

[14] X.M. Zhang, Q.L. Han, Int. J. Robust Nonlinear Control **19** (2009) 1922-1930

[15] X.L. Zhu, G.H. Yang, Int. J. Robust Nonlinear Control **20** (2010) 596-606

[16] S.B. Stojanovic, D.LJ. Debeljkovic, I. Mladenovic, Srb. J. Electr. Eng. **5** (2008) 191-198

[17] E. Fridman, U. Shaked, Eur. J. Control **11** (2005) 29-37

[18] E.K. Boukas, Math. Probl. Eng. **2006** (2006) 1-11

[19] K.F. Chen, I.K. Fong, P.I. Mech. Eng. I - J. Sys. **222** (2008) 493-500

[20] E. Fridman, U. Shaked, Eur. J. Control **11** (2005) 29-37

[21] E. Fridman, U. Shaked, Int. J. Control **78** (2005) 235-246

[22] H, Gao, T. Chen, IEEE Trans. Autom. Control **52** (2007) 328-334

[23] H. Gao, J. Lam, Y. Wang, IEE P. - Contr. Theor. Ap. **151** (2004) 691-698

[24] X. Jiang, Q.L. Han, X.H. Yu, Proc. American Control Conf., New Orleans, USA (2005) 2817-2822

[25] Y.S. Lee, W.H. Kwon, 15th IFAC World Congr., Barcelona, Spain **15** (2002), Paper ID 2378

[26] V. Leite, M. Miranda, Math. Probl. Eng. **2008** (2008) Article ID 876509, 15 pages, doi:10.1155/2008/875609, http://www.hindawi.com/journals/mpe/2008/875609/cta/

[27] X.G. Liu, R.R. Martin, M. Wu, M.L. Tang, IEE P. - Contr. Theor. Ap. **153** (2006) 689-702

[28] S. Xu, J. Lam, Y. Zou, Asian J. Control **7** (2005) 344-348

SRETEN B. STOJANOVIC[1]
DRAGUTIN LJ. DEBELJKOVIC[2]

[1]Univerzitet u Nišu, Tehnološki fakultet, Leskovac
[2]Univerzitet u Beogradu, Mašnski fakultet, Katedra za kontrolno inženjerstvo, Beograd

NAUČNI RAD

ANALIZA STABILNOSTI DISKRETNIH SISTEMA SA VREMENSKI PROMENLJIVIM KAŠNJENJEM KOJA UZIMA U OBZIR IZNOS KAŠNJENJA

Razmatrana je stabilnost diskretnih sistema sa vremenski promenljivim kašnjenjem. Koristeći odgovarajuću transformaciju modela originalnog sistema, izvedeni su dovoljni uslovi stabilnosti koji zavise od vremenskog kašnjenja. Kriterijumi stabilnosti su iskazani u obliku LMI i zavise od minimalne i maksimalne vrednosti kašnjenja. Pokazano je da ovi kriterijumi stabilnosti poseduju približno istu konzervativnost kao i postojeći kriterijumi iz literature, ali su u matematičkom smislu jednostavniji. Dat je numerički primer kojim se ilustruje upotrebljivost izvedenih rezultata.

Ključne reči: sistemi sa kašnjenjem; sistemi sa promenljivim kašnjenjem datim u obliku intervala; asimptotska stabilnost; stabilnost zavisna od kašnjenja; Ljapunova metoda.