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CHEMICAL REACTION EFFECTS ON VERTICAL OSCILLATING PLATE WITH VARIABLE TEMPERATURE

A free convective flow of a viscous incompressible flow past an infinite vertical oscillating plate with variable temperature and uniform mass diffusion is presented here, taking into account the homogeneous chemical reaction of the first-order. The plate temperature is raised linearly with respect to time, and the concentration level near the plate is raised to C'_w . An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity and concentration are studied for different parameters like phase angle, chemical reaction parameter, Schmidt number and time. It is observed that the velocity increases with the decrease of the phase angle ωt or a chemical reaction parameter.

Key words: chemical reaction; oscillating; vertical plate; heat and mass transfer.

Diffusion rates can be tremendously altered by chemical reactions. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in the solution. In most chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the first-order if the rate of the reaction is directly proportional to concentration. In many chemical engineering processes there is a chemical reaction between a foreign mass and the fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production.

Chambre and Young [1] have analyzed a first-order chemical reaction near a stationary horizontal plate using a similarity transformation. Ramanamurthy and Govindarao [2] have made an analysis of the flow around a cylindrical catalyst pellet with the first-order chemical reaction. Apelblat [3] studied an ana-

lytical solution for mass transfer with a chemical reaction of the first order. Das *et al.* [4] have studied the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer using Laplace transform technique. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [5]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [6]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar *et al.* [7]. The governing equations are tackled using the Laplace transform solution.

The study of unsteady flow past an infinite vertical oscillating plate with prescribed variable temperature and uniform mass diffusion is not studied in the literature. Hence, it is proposed to study the effects of a homogeneous chemical reaction of the first order and free convection on the oscillating infinite vertical plate with variable temperature and uniform mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of an exponential and complementary error function.

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Governing equations

The first-order chemical reaction effects on unsteady flow of a viscous incompressible fluid past an infinite vertical oscillating plate with variable temperature and uniform mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first-order chemical reaction between the diffusing species and the fluid. The x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, the plate and the fluid are of the same temperature, T_∞ , and concentration C'_∞ . At time t the plate starts oscillating in its own plane with frequency ω and the temperature of the plate is raised linearly with respect to time while the concentration level near the plate is raised to C'_w . Then, by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y'^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0: & \quad u = 0, & \quad T = T_\infty, & \quad C' = C'_\infty \text{ for all } y' \\ t' > 0: & \quad u = u_0 \cos \omega t', & \quad T = T_\infty + (T_w - T_\infty) A t', & \quad C' = C'_w \text{ at } y = 0 \\ & \quad u = 0, & \quad T \rightarrow T_\infty, & \quad C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \tag{4}$$

where:

$$A = \frac{u_0^2}{\nu}$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, y = \frac{y' u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3} \end{aligned} \tag{5}$$

$$Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, K = \frac{\nu K_1}{u_0^2}, \omega = \frac{\omega' \nu}{u_0^2}$$

in Eqs. (1)-(4), leads to:

$$\frac{\partial U}{\partial t} = Gr\theta + Gc C + \frac{\partial^2 U}{\partial y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional form are:

$$\begin{aligned} U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } y, t \leq 0 \\ t > 0: U = \cos \omega t, \quad \theta = t, \quad C = 1, \quad \text{at } y = 0 \\ U = 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \tag{9}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for a hydrodynamic flow field in the presence of the first-order chemical reaction.

Solution procedure

The dimensionless governing equations (6) to (8), a subject to the boundary conditions (9), are solved by usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t \left[(1 + 2\eta^2 Pr) \operatorname{erfc}(\eta\sqrt{Pr}) - 2\eta\sqrt{\frac{Pr}{\pi}} \exp(-\eta^2 Pr) \right] \tag{10}$$

$$\begin{aligned} C = \frac{1}{2} (\exp(2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \\ + \exp(-2\eta\sqrt{KtSc}) \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})) \end{aligned} \tag{11}$$

$$\begin{aligned} U = \frac{\exp(i\omega t)}{4} \left[\frac{\exp(2\eta\sqrt{i\omega t}) \operatorname{erfc}(\eta + \sqrt{i\omega t})}{+ \exp(-2\eta\sqrt{i\omega t}) \operatorname{erfc}(\eta - \sqrt{i\omega t})} \right] + \\ + \frac{\exp(-i\omega t)}{4} \left[\frac{\exp(2\eta\sqrt{-i\omega t}) \operatorname{erfc}(\eta + \sqrt{-i\omega t})}{+ \exp(-2\eta\sqrt{-i\omega t}) \operatorname{erfc}(\eta - \sqrt{-i\omega t})} \right] + \\ + \operatorname{derfc}(\eta) + \end{aligned}$$

$$\begin{aligned} + \frac{ct^2}{6} \left[\frac{(3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2)}{- (3 + 12\eta^2 Pr + 4\eta^4 Pr^2) \operatorname{erfc}(\eta\sqrt{Pr})} \right] - \\ + \frac{\eta\sqrt{Pr}}{\sqrt{\pi}} (10 + 4\eta^2 Pr) \exp(-\eta^2 Pr) \end{aligned} \tag{12}$$

$$-\frac{d \exp(at)}{2} \left[\frac{\exp(2 \eta \sqrt{at}) \operatorname{erfc}(\eta + \sqrt{at})}{\exp(-2 \eta \sqrt{at}) \operatorname{erfc}(\eta - \sqrt{at})} \right] -$$

$$-\frac{d}{2} \left[\frac{\exp(2 \eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} + \sqrt{Kt})}{\exp(-2 \eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{Kt})} \right] +$$

$$+\frac{d \exp(at)}{2} \left[\frac{\exp(-2 \eta \sqrt{Sc(a+K)t}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{(K+a)t})}{\exp(2 \eta \sqrt{Sc(a+K)t}) \operatorname{erfc}(\eta \sqrt{Sc} + \sqrt{(K+a)t})} \right]$$

where:

$$\eta = \frac{y}{2\sqrt{t}}$$

and

$$a = \frac{KSc}{1-Sc}, c = \frac{Gr}{Pr-1}, d = \frac{Gc}{a(1-Sc)}$$

In order to get the physical insight into the problem, the numerical values of U have been computed from Eq. (12). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$\operatorname{erf}(a + ib) = \operatorname{erf}(a) + \frac{\exp(-a^2)}{2a\pi} [1 - \cos(2ab) + i \sin(2ab)] +$$

$$+\frac{2 \exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-n^2/4)}{n^2 + 4a^2} [f_n(a, b) + i g_n(a, b)] + \varepsilon(a, b)$$

where:

$$f_n = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab)$$

and

$$g_n = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab)$$

$$|\varepsilon(a, b)| \approx 10^{-16} |\operatorname{erf}(a + ib)|$$

RESULTS AND DISCUSSION

In order to get a physical view of the problem, numerical calculations are carried out for different values of the phase angle, chemical reaction parameter, Schmidt number and time. The purpose of the calculations given here is to assess the effect of different ωt , K , Sc and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of an exponential and complementary error function.

The velocity profiles for different phase angles ($\omega t = 0, \pi/4, \pi/3, \pi/2$), $K = 2$, $Gr = 2$, $Gc = 2$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.2$ are shown in Figure 1. It is observed

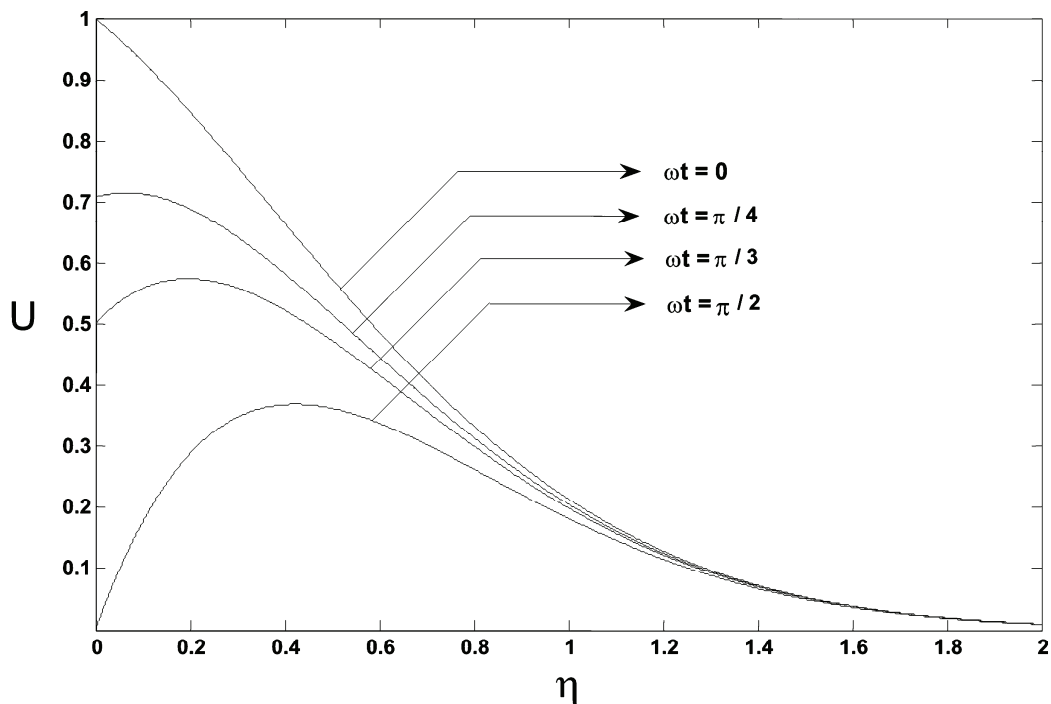


Figure 1. Velocity profiles for different values of ωt .

that the velocity increases with a decrease of the phase angle ωt .

Figure 2 illustrates the effect of the velocity for different values of the reaction parameter ($K = 0.2, 5$ and $10, \omega t = \pi/3, Gr = 5, Gc = 5, Sc = 0.6, Pr = 7.0$ and $t = 0.2$). The trend shows that the velocity increases with a decrease of a chemical reaction parameter. The effect of velocity profiles for different times

($t = 0.2, 0.3$ and $0.4, K = 2, \omega t = \pi/3, Gr = 2, Gc = 2, Sc = 0.6$ and $Pr = 7.0$) is shown in Figure 3. In this case, the velocity increases gradually with increasing the values of time t .

Figure 4 demonstrates the effect of concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5$ and $10, Sc = 0.6$ and time $t = 0.4$). It is observed that the concentration

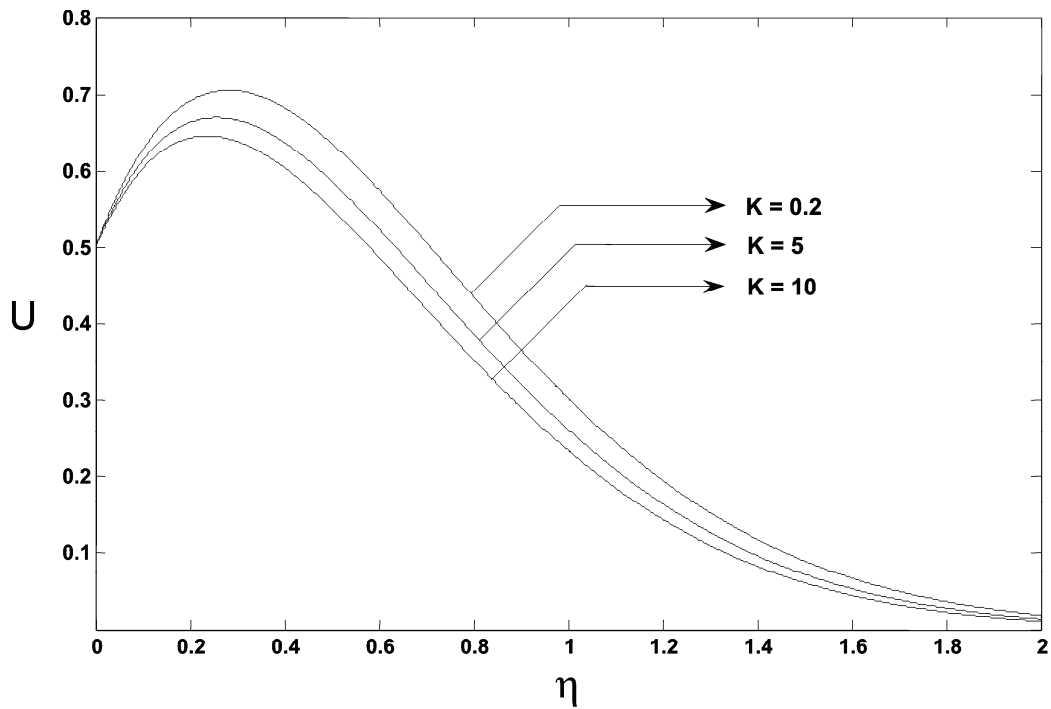


Figure 2. Velocity profiles for different values of K .

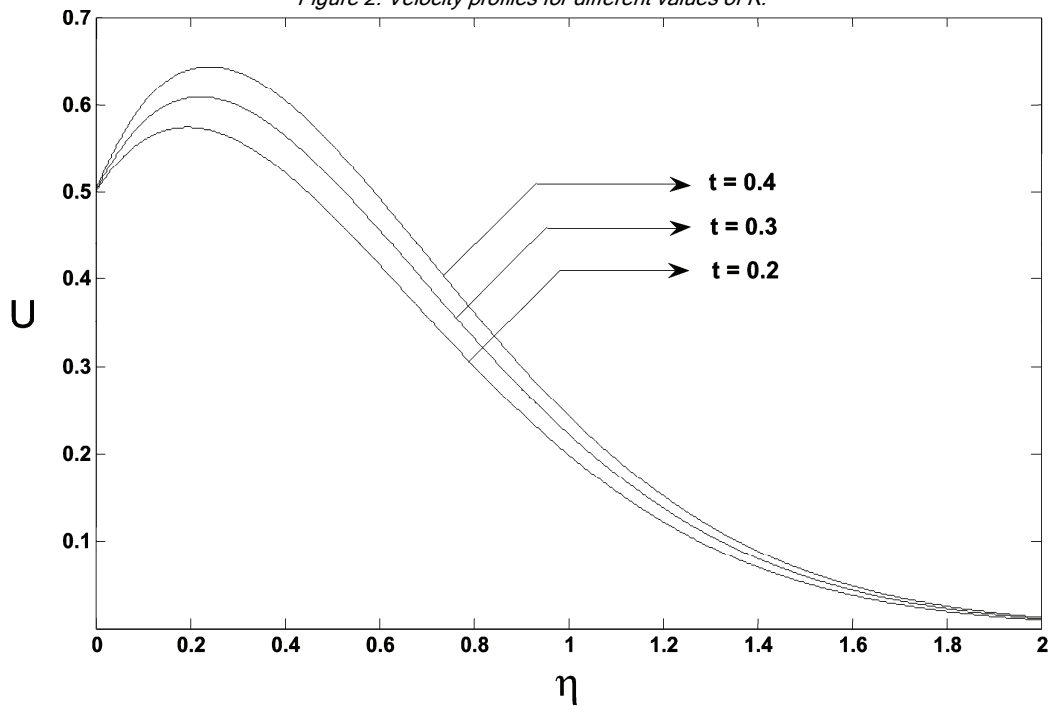


Figure 3. Velocity profiles for different values of t .

increases with a decrease of a chemical reaction parameter. The concentration profiles for different Schmidt number ($Sc = 0.16, 0.3, 0.6$ and 2.01 , $K = 0.2$ at time $t = 0.2$) are shown in Figure 5. The effect of the concentration is important in a concentration field. The profiles have a common feature that the concentration decreases in a monotone fashion from the surface to

a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing the values of the Schmidt number.

The temperature profiles calculated for different values of time ($t = 0.2, 0.4, 0.6$ and 1) are shown in Figure 6. for water ($Pr = 7.0$). The effect of a thermal radiation parameter is important in temperature pro-

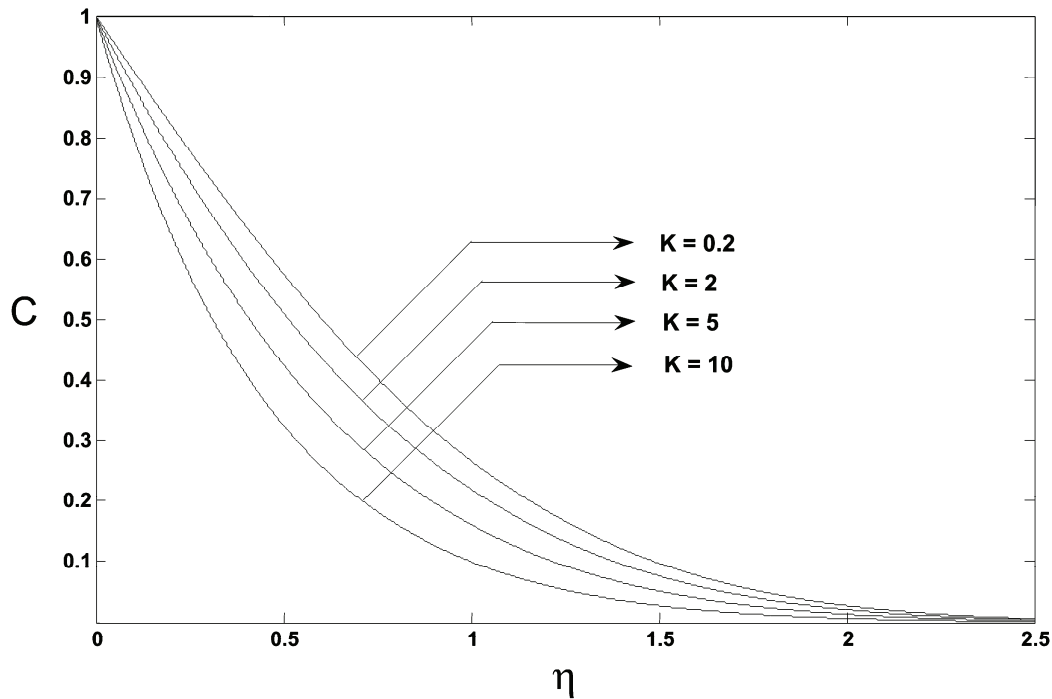


Figure 4. Concentration profiles for different values of K .

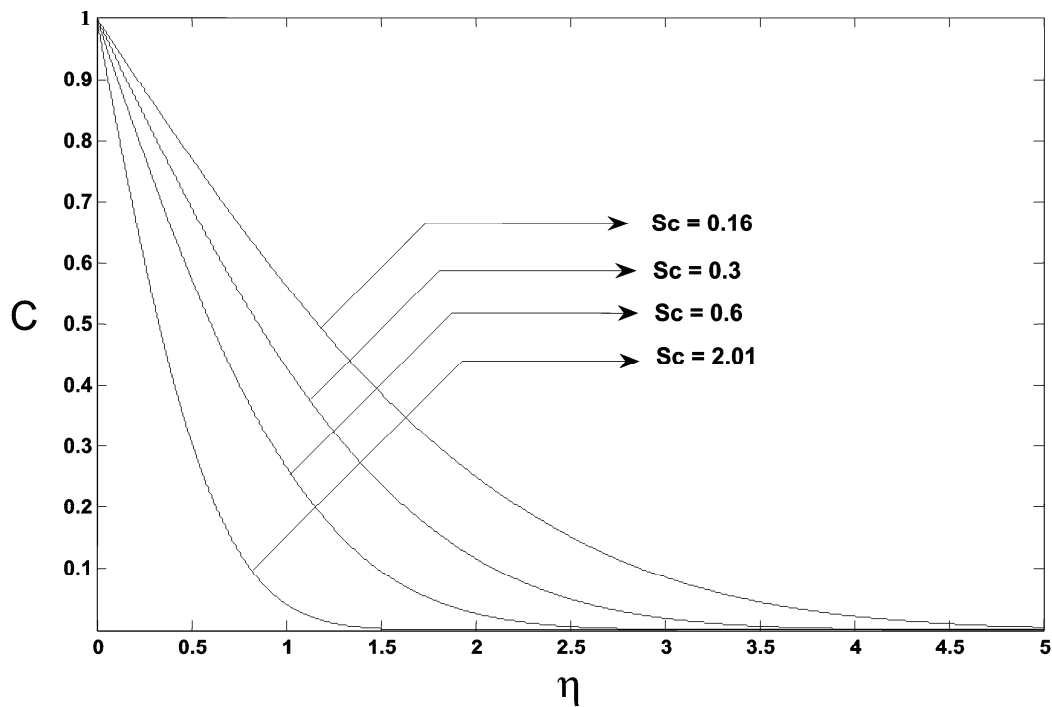


Figure 5. Concentration profiles for different values of Sc .

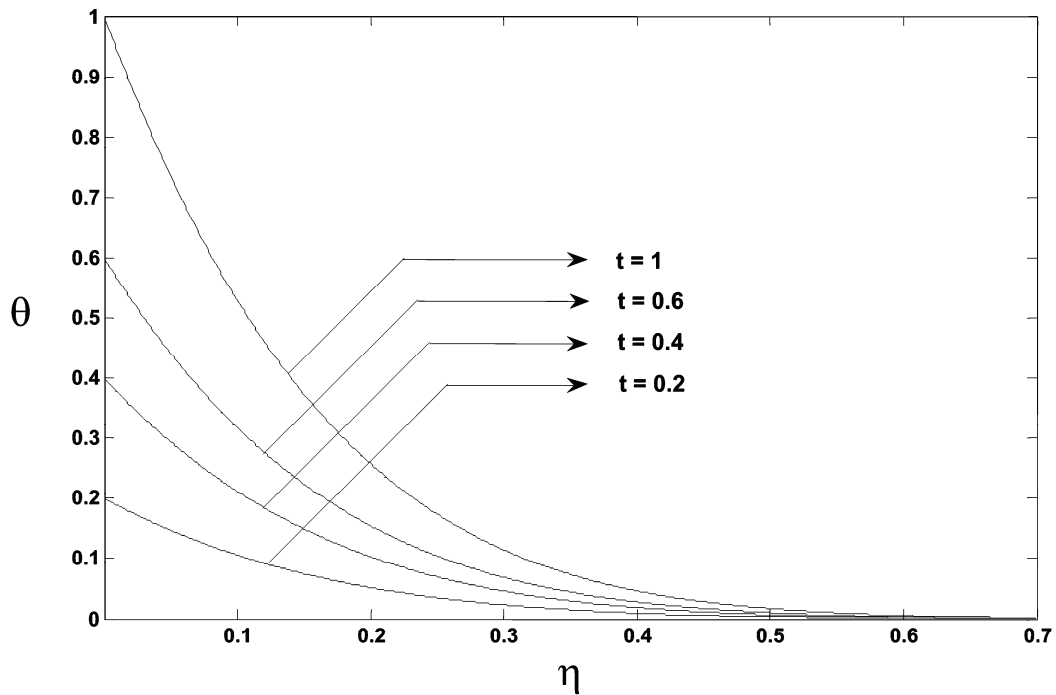


Figure 6. Temperature profiles for different values of t .

files. It is observed that the temperature increases with increasing the time t .

CONCLUSIONS

The first-order chemical reaction effects on the unsteady flow past an infinite vertical oscillating plate and in the presence of variable temperature and a uniform wall concentration is studied. The dimensionless equations are solved using the Laplace transform technique. The effect of velocity, temperature and concentration for different physical parameters like ωt , K , Sc and t are studied. It is observed that the velocity increases with decreasing a phase angle ωt and a chemical reaction parameter K . The trend is just reversed with respect to time t . It is also observed that

the concentration increases with a decrease of the Schmidt number or a chemical reaction parameter.

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NAUČNI RAD

UTICAJI HEMIJSKE REAKCIJE NA VERTIKALNU OSCILUJUĆU PLOČU SA PROMENLJIVOM TEMPERATUROM

U ovom radu je predstavljeno strujanje viskoznog nestišljivog fluida oko beskonačne vertikalne ploče prirodnom konvekcijom pri promenljivoj temperaturi i uniformnoj difuziji, u prisustvu homogene hemijske reakcije prvog reda. Temperatura ploče raste linearno sa vremenom, a koncentracija blizu ploče se povećava do C_w^ . Tačno rešenje bezdimenzionih jednačina je dobijeno metodom Laplasove transformacije, kada ploča harmonijski osciluje u sopstvenoj ravni. Uticaji brzine fluida i koncentracije ispitani su za različite parametre, kao što su: fazni ugao, parametri hemijske reakcije, Šmitov broj i vreme. Uočeno je da brzina raste sa opadanjem faznog ugla ωt ili parametra hemijske reakcije.*

Ključne reči: hemijska reakcija; oscilovanje; vertikalna ploča; prenos toplote i mase.