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SINGULAR SYSTEM THEORY APPLIED TO THE EVAPORATOR DYNAMICS OF A ONCE – THROUGH SUBCRITICAL STEAM GENERATOR: THE DIFFERENTIAL DISCRETE MATHEMATICAL MODELING BASED APPROACH

A dynamic thermal–hydraulic mathematical model of the evaporator dynamics of a once – through sub critical steam generator was derived and presented. This model allows the investigation of evaporator dynamics including its transient responses.
The evaporator was considered as part of a three–section (economizer, evaporator and super–heater) model with time varying phase boundaries and was described by a set of linearized discrete – difference equations which, with some other algebraic equations, constituted a closed system of equations possible for exact computer solution.
This model was derived using the fundamental equations of mass, energy and momentum balance.
For the first time, a discrete differential approach was applied in order to investigate such complex, two phase processes.
Namely, this approach allows one to escape from the model of this process usually described by a set of partial differential equations and enables one, using this method, to simulate evaporator dynamics in an extraordinarily simple way.
In the current literature this approach is sometimes called physical discretization.

Key words: Mathematical modelling, Distributed process dynamics, Heat exchangers, Evaporators.

PROCESS DESCRIPTION

The once – through subcritical steam generator considered in this analysis is a typical one of those used in gas–cooled nuclear power plants. It contains a large number of differently located metal tubes. Some of them can be treated as counterflow and some as crossflow heat exchangers. In this paper the latter case is of particular interest.

The focus of this investigation is a classical steam generator evaporator with time – varying boundaries. In that sense, we discuss displacements of the evaporation beginning and ending coordinate within the formal construction boundaries of evaporator.

Hot gas (primary coolant) is circulated in such a manner that it crosses perpendicularly over tube surfaces, transferring thermal energy from the primary coolant to the secondary coolant (homogeneous steam vapour) through the tube walls.

The model of the evaporating zone is based on the performance of a typical tube, schematically shown in Figure 1.

The evaporator section contains a water–steam mixture. To describe its dynamics two cases may be considered. In the case of complete evaporation, saturated water enters the evaporator, enters the evaporator tube and leaves it as saturated steam, Figure 1.

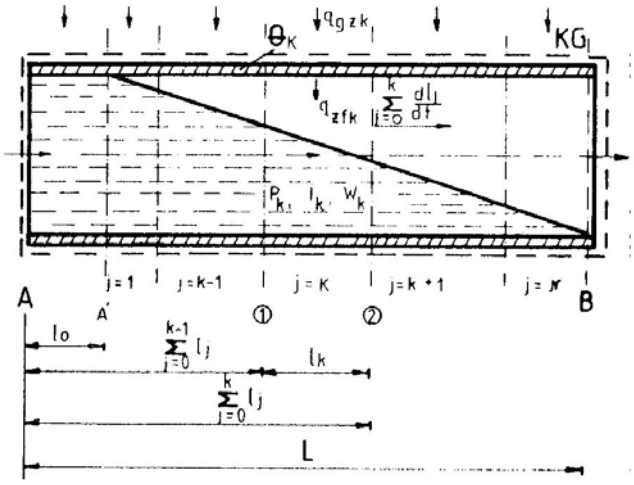

Figure 1

The exchange of thermal energy results in the cooling of the hot gas and the production of saturated steam on the tube side.

Here, the idea of physical discretatization is extended to the continuous case of a three–section model of a whole once– through steam generator with time – varying phase boundaries, first presented in Ray, Bowman (1976).

Process model adoption

Mode A corresponds to a steady state regime, containing only boiling water.

Under some – arbitrary disturbance displacements of the evaporation beginning coordinate (mode A) are

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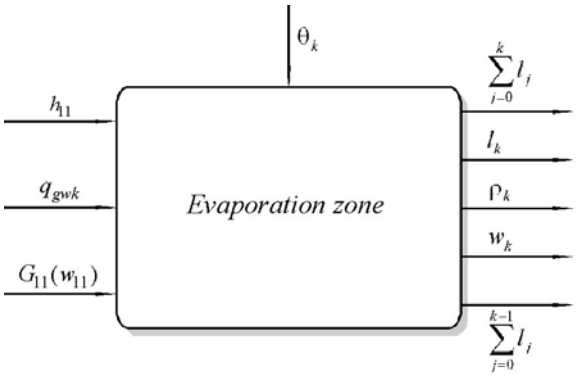

Figure 2

moved to a new position A', whereby starting from that point, a process begins in the evaporating zone.

A typical tube, representing an evaporator with fixed length, is partitioned into N cells with an accompanying index[1], successively possessing greater volume, which originates because the control volume under consideration thatis moving through the evaporator, possesses constant mass.

The length of each cell was allowed to vary with time.

The non–homogeneous field within every cell is defined by three modes, as follows:

Two boundary cross sections, denoted by: $(\cdot)_{1k}$, $(\cdot)_{2k}$ and a central one marked by $(\cdot)_k$.

In the initial steady state regime, the whole length of the evaporator zone gave L.

Mode B corresponds to the displacements of the evaporation ending coordinate defined for the evaporating zone.

Figure 2 shows evaporating zone as a transfer element with defined control, disturbance and output variables.

General assumptions

The fundamental assumption of this paper was that the distributed parameter process can be represented by a lumped difference discrete model using time–varying control volumes.

Moreover, variations of the thermodynamic properties of the two–phase mixture and the effect of relative velocity with respect to the moving phase cell boundaries were encountered in this investigation.

In addition, the following basic assumptionms were made:

Assumption. Uniform fluid properties over any cross–section.

Assumption. Uniform and independent heat fluxes across the tube wall of the evaporator for each cell.

Assumption. The two–phase mixture is always in the state of thermodynamic equilibrium, so these two components form a homogeneous mixture. The relative velocity of the vapour phase with respect to the water phase can be neglected.

Assumption. The pressure drop over the whole length of the evaporator can also be neglected:

$$p_{k-1}(t) = p_k(t) = p(t) = \text{const.} \quad (1)$$

Assumption. The temperature of the saturated vapour and the temperature of the tube walls are practically equal.

Assumption. The gas specific heat rate is constant along the length of the evaporator, e.g.:

$$q_{gwk}(t) = q_{qw(k-1)}(t) = q_{qw}(t). \quad (2)$$

Assumption. The heat conduction between the neighbour cells and throughout the tube wall can be neglected.

Assumption. There is an infinite thermal conductivity in the radial and no conductivity in the longitudinal tube direction.

Assumption. The density and the specific heat of the tube walls are constant.

Assumption. The evaporator geometric characteristics are constant and uniform along its length.

The justifications and consequences that follow from adopted assumptions can be found in Debeljković (1984) and for the sake of brevity are omitted here.

MAIN RESULT

Mathematical model process description

The thermal–hydraulic transients of the evaporator, as the crucial part of every steam generator, are essential in the study of the overall system performances of both nuclear and fossil – fueled power plants, as well as for design purposes of their appropriate control units and systems.

The basic model equations were derived from the integral forms of the fundamental equations of the conservation of mass, energy and momentum through the control volume approach.

The closed form of the system equations can be achieved if the necessary number of algebraic equations are appended to the basic balance equations.

Development of model equations

The fundamental conservation equations for the second coolant – two phase mixture for any[2] k–th cell are as follows[3]:

$$\frac{d(\rho_k l_k)}{dt} = \rho_{1k} w_1 k - \rho_{2k} w_{2k} - \rho_1 k \sum_{j}^{k-1} \frac{dl_j}{dt} + \rho_{2k} \sum_{j}^{k} \frac{dl_j}{dt} \quad (3)$$

[1] Index denotes the serial number of the cell and increases from the first cell up to the last one.

[2] With the exception of $k=1$ and $k=N$.

[3] It should be emphasised that the expression discrete is used to underline discretization of the spatial coordinate

$$\frac{d(\rho_k l_k h_k)}{dt} = \rho_{1k} w_{1k} h_{1k} - \rho_{2k} w_{2k} h_{2k} - \rho_{1k} h_{1k} \sum_{j}^{k-1} \frac{dl_j}{dt}$$

$$+ \rho_{2k} h_{2k} \sum_{j}^{k} \frac{dl_j}{dt} + \frac{d(\rho_k l_k)}{dt} + \frac{1}{A_u} Q_{wfk} \quad (4)$$

$$\frac{d(\rho_k l_k w_k)}{dt} = \rho_{1k} w_{1k}^2 - \rho_{2k} w_{2k}^2 - \frac{\zeta}{2 d_n} l_k w_k$$

$$- \rho_{1k} w_{1k} \sum_{j}^{k-1} \frac{dl_j}{dt} + \rho_{2k} w_{2k} \sum_{j}^{k} \frac{dl_j}{dt} \quad (5)$$

For the evaporator, equations (3–5) represent equations of mass, energy and momentum conservation for the *k*–th cell, being the control volume under consideration.

The energy equation in the tube wall is as follows:

$$\rho_w c_w (A_o - A_i)\, \pi \frac{d(\theta_{wk} l_k)}{dt} = \frac{1}{A_{ok}} Q_{gwk} - \frac{1}{A_{ik}} Q_{wfk} \quad (6)$$

Addends in the form of sums in the preceding equations originate from the fact that there exists a relative velocity of the two phase mixture with respect to the moving cell boundaries, Figure 1.

$$\frac{dl_k}{dt} = \frac{1}{T_v} l_k\,, \quad \sum_{j=1}^{N} l_j(t) = L(t) \quad (7)$$

$$l_k = \sum_{j}^{k} l_j - \sum_{j}^{k-1} l_j\,, \quad \frac{dl_k}{dt} = \sum_{j}^{k} \frac{dl_j}{dt} - \sum_{j}^{k-1} \frac{dl_j}{dt} \quad (8)$$

The law of volume increase of a particular fluid mass element within the *k*–th cell is expressed by (7), as well as the possible non–stationary behaviour of the full length of the evaporator.

Based on the control volume definition and simple geometric relationships between neighbour cells equation (8) follows.

The time constant T_e, is known as the evaporation time, and can be found in Profos (1962).

It has been shown, Hebrik (1972), that the heat transfer coefficients in the boiling regime are usually very high. The wall temperature is therefore nearly equal to the steam saturation temperature, so one can have:

$$\theta_{wk} = \theta''_{fk}\,, \quad \theta''_{fk} = \theta''_{fk}(p_k)\,, \quad \frac{d\theta_{wk}}{dt} = \left(\frac{d\theta_{wk}}{dp_k}\right)_N \frac{dp_k}{dt} \quad (9)$$

The saturation temperature θ''_f can only vary with pressure.

The thermodynamic equations of state are used to express the pressure as the function of density and entalphy:

$$\rho_k = \rho_k(p_k, h_k)\,, \quad \frac{d\rho_k}{dt} = \left(\frac{\partial \rho_k}{\partial p_k}\right)_N \frac{dp_k}{dt} + \left(\frac{\partial \rho_k}{\rho h_k}\right)_N \frac{dh_k}{dt} \quad (10)$$

and can be efficiently used in the evaporation process.

Besides the before–mentioned conservation laws, Newtonžs second law applied to the control volume must also be used and is given as:

$$Q_{wfk} = \alpha_{wfk} A_{ik} (\theta_{wk} - \theta_{fk})\,, \quad \alpha_{wfk} = \alpha_{wfk}(w_k) \quad (11)$$

$$A = d^2\pi/4\,, \quad A_{ik} = d_i \pi l_k\,, \quad A_{ok} = d_o \pi l_k \quad (12)$$

$$(\cdot)_k = \frac{1}{2}\left((\cdot)_{1k} + (\cdot)_{2k}\right) \quad (13)$$

The connection between the physical variables in the boundary cross sections and the central one is given by eq. (13).

Eqs. (3–8) complete the set of non–linear differential – discrete equations necessary to describe the non–stationary behavior of a water–steam mixture in the evaporator for specific and initial conditions.

Using eqs. (9–13), this set of equations can be simplified and expressed into its general form as follows:

$$\frac{dp_k}{dt} = f_1\left(\frac{dh_k}{dt}, \frac{d\sum_j^k l_j}{dt}, \frac{d\sum_j^{k-1} l_j}{dt}, p_k, h_k, h_{1k}, w_k, w_{1k}, \sum_j^k l_j, \sum_j^{k-1} l_j\right) \quad (14)$$

$$\frac{dh_k}{dt} = f_2\left(\frac{dp_k}{dt}, \frac{d\sum_j^k l_j}{dt}, \frac{d\sum_j^{k-1} l_j}{dt}, p_k, h_k, h_{1k}, \ldots\ldots, \ldots\ldots w_k, w_{1k}, \sum_j^k l_j, \sum_j^{k-1} l_j, \theta_k, Q_{gwk}\right) \quad (15)$$

$$\frac{dw_k}{dt} = f_3\left(\frac{d\sum_j^k l_j}{dt}, \frac{d\sum_j^{k-1} l_j}{dt}, p_k, h_k, h_{1k}, \ldots\ldots, \ldots\ldots w_k, w_{1k}, \sum_j^k l_j, \sum_j^{k-1} l_j\right) \quad (16)$$

$$\frac{d\sum_j^k l_j}{dt} = f_4\left(\frac{dp_k}{dt}, \frac{d\sum_j^{k-1} l_j}{dt}, p_k, w_k, \sum_j^k l_j, Q_{gwk}\right) \quad (17)$$

$$\frac{d\sum_j^{k-1} l_j}{dt} = f_5\left(\frac{d\sum_j^k l_j}{dt}, \sum_j^k l_j, \sum_j^{k-1} l_j\right) \quad (18)$$

$$k = 1, 2, \ldots, N. \quad (19)$$

The functions f_i, $i = 1, 2, \ldots, 5$ are non–linear.

If one defines the relative deviations of all the introduced variables in the following manner:

$$\overline{\Delta(\cdot)_k} = \frac{(\cdot)_k - (\cdot)_{kN}}{(\cdot)_{kN}} \quad (20)$$

and by defining the control, disturbance and state variables:

$$\overline{\Delta p_k}(t) = x_1(t), \quad \overline{\Delta Q_{gwk}}(t) = u_{k1}(t),$$

$$\overline{\Delta h_k}(t) = x_2(t), \quad \overline{\Delta w_0}(t) = u_2(t),$$

$$\mathbf{x}_k(t) = [x_{k1} \ldots x_{k5}]^T, \quad \overline{\Delta w_k}(t) = x_3(t) \quad (21)$$

$$\overline{\Delta h_0}(t) = z_1(t), \quad \mathbf{u}_k(t) = [u_{k1} \quad u_2]^T,$$

$$\sum_j^k \overline{\Delta l_j}(t) = x_4(t), \quad \overline{\Delta\theta_k}(t) = z_{k2}(t)$$

$$\mathbf{z}_k(t) = [z_1 \quad z_{k2}]^T, \quad \sum_j^{k-1} \overline{\Delta l_j}(t) = x_5(t),$$

and under the assumption that all the conditions necessary to carry out the linearization, one may obtain the mathematical model of the evaporation process in the k–th cell:

$$E_k\mathbf{x}_k(t) = N_k^1\mathbf{x}_k(t) - 2N_k^2\mathbf{x}_{k-1}(t) + \\ + N_k^2\mathbf{x}_{1,k-1}(t) + M_k\mathbf{u}_k(t) + H_k\mathbf{z}_k(t) \quad (22)$$

The used superscripts are only used to introduce adequate notation and do not have any mathematical meaninig.

It should be noted that the transformation of the state vector $\mathbf{x}_{1k}$ in eq. (22) was carried out using the following relation:

$$(\cdot)_{1k} = (\cdot)_{2,k-1} = 2(\cdot)_{k-1} - (\cdot)_{1,k-1} \quad (23)$$

The structure of the matrices defined in eq. (22) is given as:

$$N_k^1 = \begin{bmatrix} n_{11}^k & n_{12}^k & n_{13}^k & n_{14}^k & n_{15}^k \\ n_{21}^k & n_{22}^k & n_{23}^k & n_{24}^k & n_{25}^k \\ n_{31}^k & n_{32}^k & n_{33}^k & n_{34}^k & n_{35}^k \\ n_{41}^k & 0 & n_{43}^k & n_{44}^k & n_{45}^k \\ 0 & 0 & 0 & n_{54}^k & n_{55}^k \end{bmatrix}, \quad E_k = \begin{bmatrix} 1 & e_{12}^k & 0 & e_{14}^k & e_{15}^k \\ e_{21}^k & 1 & 0 & e_{24}^k & e_{25}^k \\ 0 & 0 & 1 & e_{34}^k & e_{35}^k \\ 0 & 0 & 0 & 1 & e_{45}^k \\ 0 & 0 & 0 & e_{54}^k & 1 \end{bmatrix}, \quad (24)$$

$$N_k^2 = \begin{bmatrix} 0 & n_{12}^{k-1} & n_{13}^{k-1} & 0 & 0 \\ 0 & n_{22}^{k-1} & n_{23}^{k-1} & 0 & 0 \\ 0 & n_{32}^{k-1} & n_{33}^{k-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad M_k = \begin{bmatrix} 0 & 0 \\ m_{21}^k & 0 \\ 0 & 0 \\ m_{41}^k & 0 \\ 0 & 0 \end{bmatrix}, \quad H_k = \begin{bmatrix} 0 & 0 \\ 0 & h_{22}^k \\ 0 & 0 \\ 0 & h_{42}^k \\ 0 & 0 \end{bmatrix}.$$

In a particular case, for k=1, the matrices N_k^2, M_k and H_k assume the following structure:

$$N_1^2 = [0] \quad (25a)$$

$$M_1 = \begin{bmatrix} 0 & 3n_{13}^1 \\ m_{21}^1 & 3n_{23}^1 \\ 0 & 3n_{33}^1 \\ m_{41}^1 & 0 \\ 0 & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 3n_{12}^1 & 0 \\ 3n_{22}^1 & h_{22}^1 \\ 3n_{32}^1 & 0 \\ 0 & h_{42}^1 \\ 0 & 0 \end{bmatrix} \quad (25b)$$

with the state space representation of the first cell:

$$E_1\mathbf{x}_1(t) = N_1^1\mathbf{x}_1(t) + M_1\mathbf{u}(t) + H_1\mathbf{z}(t) \quad (26)$$

Particular expressions for the matrix elements are given in Debeljkovic (2002, 2004), and are of some interest here.

If one defines the control, disturbance, output and state variables for the whole process occurring in evaporator:

$$\mathbf{x}(t) = [x_1 \quad x_2 \quad \ldots x_N]^T,$$

$$\mathbf{u}(t) = [u_{k1} \quad u_2]^T, \quad \mathbf{z}(t) = [z_1 \quad z_{k2}]^T \quad (27)$$

$$\mathbf{x}_i(t) = [0000x_{i51}\ldots0000\ldots x_{i1N} \quad \mathbf{0} \quad x_{i3N} \quad x_{i4N} \quad \mathbf{0}]^T$$

the matrix model should be in the following form:

$$E\mathbf{x}(t) = N\mathbf{x}(t) + N\,\mathbf{x}_{(1)}(t) + M\mathbf{u}(t) + H\mathbf{z}(t), \quad (28)$$

$$\mathbf{y}(t) = C\mathbf{z}(t), \quad (29)$$

where:

$$E = diag\,\{E_1, E_2, \ldots, E_N\},$$

$$M = [\,M_1 \,|\, M_2 \,|\, \ldots \,|\, M_N\,]^T,$$

$$H = [\,H_1 \,|\, H_2 \,|\, \ldots \,|\, H_N\,]^T,$$

$$C = diag\,\{\,C_1, C_2, \ldots, C_N\,\},$$

$$N* = diag\,\{\,0, N_2^2, \ldots, N_N^2\,\},$$

$$C_1 = [0\,], \quad j = 2, 3, \ldots, N-1$$

$$N = \begin{bmatrix} N_1^1 & 0 & 0 & L & 0 & 0 \\ N_2^2 & N_2^1 & 0 & L & 0 & 0 \\ 0 & N_2^2 & 0 & L & 0 & 0 \\ M & M & M & 0 & M & M \\ 0 & 0 & N_N^2 & L & N_N^2 & N_N^1 \end{bmatrix} \quad (30)$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

CONCLUSION

The evaporator under consideration was considered as part of a once – through steam generator defined by a three–section (economizer, evaporator and super–heater) model with time varying phase boundaries.

In order to avoid partial differential equations, which are usually used to describe such distributed parameter processes, the method of physical discretatization was introduced, defining for the first time the concept of a time varying cell leading to the state space representation in the form of a set of linearized discrete – difference equations which, with some other algebraic equations, constitutes a closed system of equations that allow an exact computer solution.

Some difficulties can arise in the circumstances when the matrix in eq. (24) or (26) may be singular.

Such cases can be treated in a particular way, based on different approaches developed for such a class of so called singular systems, Debeljkovic (2004).

SIMULATION RESULTS AND DISCUSSION

The steady state performances of the linearized differential – difference dynamic model were tested at 10 percent rated conditions.

Some results of the evaporator simulation are presented in the form of curves representing transient responses of the single output variable – displacement of ending evaporation coordinate at the rated conditions for independent step increases in two different input variables. In each case, the input variable under study was

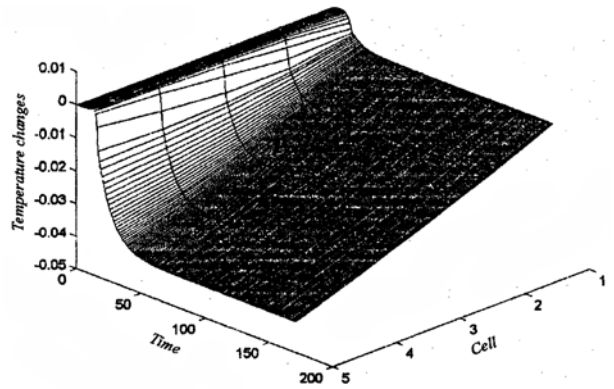

Figure 3

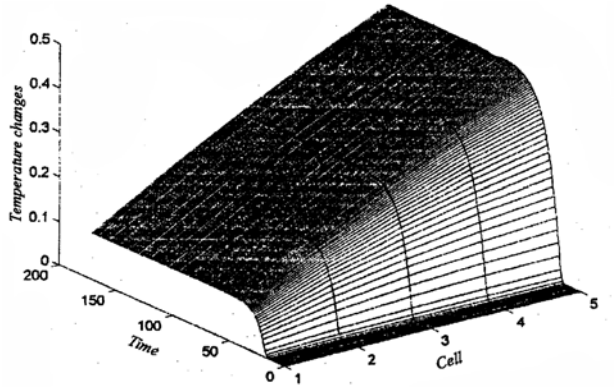

Figure 4

perturbed from its operating point by a 5–percent step increase, with the other input variable held constant.

Figure 3 presents the process transient response for a step increase in a gas specific heat flow rate.

Figure 4 shows the process transient response for a step increase in a boiling water mass flow rate.

SOME BASIC FACTS ON SINGULAR PROCESSES

Singular processes (systems) are those the dynamics of which are governed by a mixture of algebraic and differential equations.In that sense the algebrac equations represent the constraints to the solution of the differential part.

These systems are also known as a descriptor and semi–state and arise naturally as a linear approximation of systems models, or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large–scale systems, interconnected systems, economics, optimization problems, feedback systems, robotics, biology, etc.

The complex nature of the generalized state space systems causes many difficultes in analytical and numerical treatment that do not appear when systems in the normal form are considered. In this sense questions of existence, solvability, uniqueness, and smoothness are present which must be solved in a satisfactory manner.

A short and concise, acceptable and understandable explanation of all these questions may be found in the papers of Debeljkovic (2004).

Consider a linear singular process represented by:

$$E\mathbf{x}(t) = A\mathbf{x}(t)\ ,\quad \mathbf{x}(t_o) = \mathbf{x}_0\ ,$$

$$\mathbf{y}(t) = C\mathbf{x}(t) \tag{31}$$

and

$$E\mathbf{x}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)\ ,\quad \mathbf{x}(t_0) = \mathbf{x}_0$$

$$\mathbf{y}(t) = C\mathbf{x}(t) \tag{32}$$

with the matrix E possibly singular, where $\mathbf{x}(t) \in \Re^n$ is a generalized state–space vector, $\mathbf{u}(t) \in \Re^m$ is a control variable, and $\mathbf{y}(t) \in \Re^p$.

The matrices E, $A \in \Re^{nxn}$, $B \in \Re^{nxm}$ and $C \in \Re^{pxn}$ are of the appropriate dimensions and are defined over the field of real numbers.

The system given by eq. (31) is operatinig in a free and system given by eq. (32) is operating in a forced regime, i.e. some external force is applied on it. It should be stressed that, in the general case, the initial conditions for an autonomous and a system operating in the forced regime need not be the same.

System models of this form have some important advantages in comparison with models in the normal form, e.g. when E = I and an appropriate discussion can be found in Debeljkovic et al. (1996, 1996a, 1998, 2004, 2005), Debeljković (2004).

Eqs. (31–32) arise naturally in the process of modeling various physical systems, when the equations are written in the sparse form.

They have some important advantages in comparison with models in the normal form, Bajić (1992):

- the models preserve the sparsity of the system matrices
- there is a tight relation between the system physical variables and the variables in these models,
- the structure of the physical system is well reflected in the models,
- there is a great simplicity in the derivation of eqs. (31–32) and in this connection there is no necessity for the elimination of the unwanted (redundant) variables, as there is no request to build the models in the traditional form.

Models of the mentioned forms can be found in many different fields.

The Laplace transformaion of the system, given by eq. (32), under zero conditions, results in the following generalized transfer function matrix:

$$W(s) = C\,(sE - A)^{-1}\ B = C\,\frac{adj\,(sE - A)}{\det\,(sE - A)}\,B\ , \tag{33}$$

a transfer matrix of a singular system, with the characteristic equation of the form:

$$f_E(s) = \det\,(sE - A). \tag{34}$$

It can be shown that the transfer function of linear singular systems, in certain circumstances, can not be found. This problem is completely determined by the question of the possible solvability of a singular system, see Debeljković (2004).

From eq. (33), it is obvious that only a regular singular system can have such a description.

If a singular system has no transfer function, i.e. it is irregular, it may still have a general description pairing, Dziurla, Newcomb (1987), that is a description of the form:

$$R(s)\,\mathbf{Y}(s) = Q(s)\,\mathbf{U}(s) \tag{35}$$

where **Y**(s) and **U**(s) are Laplace transforms of the output and input, respectively. Since, irregular systems may have many or no solutions at all, the question arises as to whether we would meet them in practice.

Th mentioned reference shows that we indeed meet them, at least when we idealize certain systems.

Other aspects, concerning the solvability and state structure for irregular singular systems can be found in Dai (1989.a).

A practical and compact procedure for obtaining the transfer function of linear singular systems, especially for high order systems, is not based on eq (31), but some specific procedures based on a finite – series expansion for $(sE - A)^{-1}$.

The singular system is regular, when the matrix pencil (cE –A) is regular, i.e.

$$\exists c \in \Re : \det(cE - A) \neq 0, \tag{36}$$

and then solutions of eq. (31) exist, they are unique and for so–called consistent initial conditions generate smooth solutions. Moreover, the closed form of this solution is known, Dai (1989.b).

In some circumstances, it is useful to introduce the linear non–singular transformation of a system governed by eq. (33), in order to obtain the first canonical form of a linear singular system, as:

$$\mathbf{x}(t) = A_1\,\mathbf{x}_1(t) + A_2\,\mathbf{x}_2(t) \tag{37}$$

$$\mathbf{0} = A_3\mathbf{x}_1(t) + A_4\mathbf{x}_2(t)\,, \tag{38}$$

The regularity condition (36) from the system given by eqs. (37–38) reduces to the following:

$$\det\begin{pmatrix} sI & -A_1 & -A_2 \\ -A_3 & -A_4 \end{pmatrix} \neq 0\,, \tag{39}$$

which is equivalent to:

$$\det(sI - A_1)\,\det(-A_4 - A_3(sI - A_3)^{-1}A_2) \neq 0\,, \tag{40}$$

or:

$$\det A_4\,\det((sI - A_1) - A_2 A_4^{-1} A_3) \neq 0\,, \tag{41}$$

Instead of eq. (36), one can verify the following condition, Campbell (1980).

$$\aleph(A) \cap \aleph(E) = \{0\}, \tag{42}$$

i.e. $\aleph(A)$ and $\aleph(E)$ have only a trivial intersection where $\aleph(\cdot)$ denotes the null space or kernel of matrix $(\cdot)$.

Owens and Debeljkovic (1985) showed that eq. (42) is equivalent to:

$$W_{k^*} \cap \aleph(E) = \{0\}, \tag{43}$$

W_{k^*} being the subspace of consistent initial conditions.

Alternative characterizations of the regularity condition offered by other authors are also presented in the current literature.

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